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MAGNETIZATION BY ROTATION.[1]

By S. J. Barnett.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.

Thus consider a cylinder of iron, with zero magnetic moment in its initial state. If it is given an angular acceleration about its axis, each individual system, which we may suppose for simplicity to consist of a number of electrons revolving in fixed orbits with constant average velocities about an oppositely charged nucleus, will change its orientation in such a way as to contribute a minute angular momentum, and therefore a minute magnetic moment, parallel to the axis of the cylinder. This increment of angular momentum of each system is in the direction of the axis of rotation, and the corresponding increment of the magnetic moment is either in this direction or in the opposite direction according as the particles in revolution are positive or negative. If the revolving electrons are all negative, in conformity with most of the experimental evidence, the cylinder will become magnetized in the direction in which it would be magnetized by an electric current flowing around it in a direction opposite to that of the angular velocity imparted to it. This corresponds to the direction of magnetization of the earth and the sun.

§2. Preliminary experiments made at the Tulane University of Louisiana at the time this idea occurred to me appeared to show,[2] though doubtfully, a very minute effect of the sort in question, on the assumption that the revolving electrons are negative, in the case of a steel rod about

[1] Revision of papers read before the Ohio Academy of Sciences, November, 1914, and the American Physical Society, November, 1913, December, 1914, and April, 1915.

[2] S. J. Barnett, Science, 30, 1909, p. 413.

7 cm. in diameter and half a meter long driven at a speed of about 90 revolutions per second. In these experiments two approximately cylindrical and similar electromagnets were mounted with their axes parallel and approximately perpendicular to the earth's intensity. One of the cores and both of the coils were fixed while the other core was rotated by an induction motor at a distance. The two coils were connected in series through a ballistic galvanometer in such a way as to compensate for any change of flux due to alteration of the earth's intensity. Changes of flux were determined by the galvanometer throws occurring on starting and stopping the motor. Later observations made in much the same way, but with an attempt at improvement in apparatus, failed to confirm this result with any certainty; and further investigation of the subject was postponed until better facilities were available.

§3. In 1912 a similar idea was advanced by Schuster[1] in his presidential address before the Physical Society of London. He also appears to have been led to his views by considering the origin of the earth's magnetism.

Referring to other hypotheses as to the production of magnetization by rotation, and assuming, for the sake of argument, that they account for terrestrial magnetism, he shows that one of the hypotheses cannot be correct, as it would then have been easily established by common observations on rotating bodies, and that the other leads to effects too minute to be detected by the most refined observations.

He then takes up the hypothesis similar to that introduced here,—a hypothesis according to which rotation, instead of *directly determining magnetization*, "determines magnetic intensity which may or may not cause magnetization according to the nature of the body." "It is," he says, " . . . quite in accordance with our present views that every rotating body should be subject to a magnetizing force along the axis of any rotation that may be impressed upon it. If magnetization be due to a circulation of electrons within the molecules these should to some extent behave like gyrostatic compasses, setting themselves parallel to the axis of rotation of the body which contains them. Further properties of molecular constitution have, however, to be specified before we can say whether any actual magnetization results. If the electron is free,[2] the main result, I believe, would be only a magnetic precession round the axis of rotation, and in this we may find a powerful argument [for this explanation of terrestrial magnetism], because the same cause which gives us the magnetic force, also gives us the secular variation[3] . . . "

[1] A. Schuster, Proc. Phys. Soc. London, 24, 1911–12, p. 121.

[2] A hypothesis contrary to mine.

[3] In this address Schuster refers to an address made eleven years earlier in which he suggested the possibility "that terrestrial magnetism is due to the rotation of the electron round

"On the other hand, the theory [in order to account for terrestrial magnetism] would have to explain why the iron inside the earth becomes more strongly magnetized than the iron in our laboratories. There is, of course, always the possibility of some substance being subject to the effects of rotation in a much higher degree than iron. Such questions can only be settled by experiments which are now in progress." On the importance of these, and other such experiments, Schuster lays great stress.

§4. It soon occurred to me that the effect under investigation was the converse of the effect predicted and looked for by O. W. Richardson[1] in 1907–8, viz., the production of rotation by magnetization; and it became apparent that both effects were immediate consequences of an idea advanced long ago according to which a magnet must behave like a gyrostat if the Ampèreian currents consist in the motion of actual matter. This idea is due to Maxwell,[2] who constructed apparatus for experiments upon the subject as early as 1861.

In Maxwell's experiments an electromagnet was pivoted in a frame in such a way as to be free to rotate about a horizontal line through its center of mass and perpendicular to its magnetic axis. With the magnetic axis making an angle θ with the vertical, the frame was rotated at high speed about a vertical axis, and optical observations were made for a change in θ, stability having been secured by suitable adjustments of the principal moments of inertia. No change was detected, but only rough observations were possible.

My own experiment may be considered as a modification of Maxwell's, and the principal equation in the development of its theory, after that giving the relation between the angular momentum and the magnetic moment of a molecular magnet, is a special case of his equation for the torque acting to diminish the angle θ. In my experiment Maxwell's electromagnet is replaced by each of the countless multitude of molecular magnets of which the iron rod is constituted, and the total change in the orientation of all these magnets with reference to the axis of rotation of the rod is determined magnetically instead of optically.

Fig. 1.

§5. A quantitative theory of the effect will now be developed.

Consider the simplest type of molecular magnet (Fig. 1), in which a single particle with charge e and mass m revolves in a closed orbit about

the atom" as a means of explaining magnetic precession. The quotation from this earlier address, of which I first learned from the 1912 paper, would indicate that Schuster had probably in mind in 1901 the idea which he proposed definitely in 1912.

[1] O. W. Richardson, PHYS. REV., 26, 1908, p. 248.

[2] Electricity and Magnetism, § 575.

a much more massive nucleus with charge $-e$. If r denotes the radius vector, ω the angular velocity, $\alpha = \frac{1}{2}r\omega^2$ the areal velocity, μ the magnetic moment of the molecular system, and M the angular momentum due to the orbital motion, we have

$$\mu = e\alpha \quad \text{and} \quad M = mr^2\omega = 2m\alpha. \tag{1}$$

Hence

$$\frac{M}{\mu} = 2\frac{m}{e}, \quad \text{or} \quad M = 2\frac{m}{e}\mu; \tag{2}$$

so that the angular momentum is directly proportional to the magnetic moment.

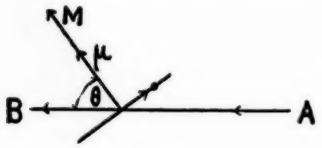

Fig. 2.

If the normal to the orbit (Fig. 2) makes an angle θ with any direction AB, the component of the magnetic moment μ in this direction is $\mu \cos\theta$, and that of the angular momentum is thus

$$M \cos\theta = 2\frac{m}{e}\mu \cos\theta.^1 \tag{3}$$

If the body of which the molecular system of Figs. 1 and 2 is a part is set into rotation about the axis AB with angular velocity Ω, the angle between the vector representing M, the angular momentum of the system due to its orbital motion, and AB will decrease, just as in the case of a gyroscope, until the torque T' on the revolving system brought into existence by this displacement is just equal to the rate of increase of its total angular momentum in the steady state when kinetic equilibrium has been attained and the vector M is tracing out a conical surface with constant semiangle θ and angular velocity Ω. The effect in this steady

[1] Assuming m/e identical for all the orbital systems, and summing over the unit volume, we have

$$\Sigma M \cos\theta = 2\frac{m}{e}\Sigma\mu \cos\theta = 2\frac{m}{e}I, \tag{4}$$

which is the formula developed by Richardson (l. c.), but by a complicated process. Here $I = \Sigma\mu \cos\theta$ is the intensity of magnetization, and $\Sigma M \cos\theta$ is the angular momentum of the electrons per unit volume, both reckoned in the direction AB.

This fundamental equation of Richardson's theory is developed in essentially this same manner, in which it was presented in the first two papers which this article reports, in a more recent paper presented by Einstein and de Haas in February and April (D. Ph. Ges., No. 8, April 30, 1915). In this paper are described experiments on soft iron by the method of resonance which appear to confirm Richardson's theory. I had myself started experiments on this effect, beginning with the ballistic method but planning to use the method of resonance if necessary. From my ballistic experiments I can state that in the case of brass no effect exists comparable with that which the simple theory indicates should exist in the case of iron. Einstein and de Haas have found the same thing true of copper. Their paper contains no reference to the previous work of Maxwell, Schuster, Richardson, or myself.

state is exactly the same as if the body were at rest and the system were acted upon by a torque $T'' = -T'$ due to an extraneous magnetic field with strength H equal to the intrinsic magnetic intensity of rotation. The complete expression for T'' is known (and can readily be shown from first principles) to be

$$T'' = -T' = -M\Omega \sin\theta - B\Omega^2 \sin\theta\cos\theta, \tag{5}$$

where B denotes the excess of the moment of inertia of the system about the axis of its orbital angular velocity ω over the moment of inertia about that diameter of the orbit making with AB the angle $90°-\theta$. If the orbit is circular

$$B = mr^2 - \tfrac{1}{2}mr^2 = \frac{M}{2\omega}. \tag{6}$$

Eliminating B and M from (5), we get

$$T'' = -\mu \sin\theta \cdot 2\frac{m}{e}\Omega\left(1 + \tfrac{1}{2}\frac{\Omega}{\omega}\cos\theta\right).^{1} \tag{7}$$

Dividing this expression by $-\mu\sin\theta$, as in the case of an ordinary magnetic field, we get the intrinsic intensity of rotation:

$$H = 2\frac{m}{e}\Omega\left(1 + \tfrac{1}{2}\frac{\Omega}{\omega}\cos\theta\right).^{2} \tag{8}$$

The values of Ω experimentally attainable are so small in comparison with ω that the second term is negligible. If the orbit is not circular we obtain for H an expression whose first term is identical with that of (8) and whose second term has the same order of magnitude as that of (8).

If we assume that e/m has the value ordinarily accepted for the negative electron in slow motion, viz., -1.77×10^7, and put $\Omega = 2\pi n$, where n is the angular velocity in revolutions per second, we obtain for the intensity per unit angular velocity

$$H/n = -7.1 \times 10^{-7}\frac{\text{gauss}}{\text{r.p.s.}}. \tag{9}$$

This is on the assumption that the negative electron alone is effective. According to this, all substances would be acted upon by precisely the same intensity for the same angular velocity.

If some or all of the positive ions also have orbital motions, proportionality with angular velocity will evidently still exist, but the coefficient of Ω will be reduced in magnitude or even changed in sign, and the intensities acting on different substances may differ for the same value of Ω.

[1] This equation, as stated above, also follows immediately from Maxwell's equation for the torque when the conditions here assumed are put in.

[2] The first term of this equation has been given previously by Einstein and de Haas (l. c.), but is incorrectly derived in their paper, equations for a *molar* magnet instead of a *molecular* magnet being employed. All the terms of their equations reduce to zero unless the body is originally magnetized.

If the influence of the negative electrons is preponderant, the number in (9) gives the *maximum* magnitude of H/n, attained when the negative electrons alone are effective.

The relation of proportionality must hold also between the angular velocity and the magnetic flux density and intensity of magnetization, which are very minute and therefore proportional to the intrinsic intensity; but these quantities will depend not only upon the intensity but also upon the material and shape of the rotating body. Thus in the case of a diamagnetic substance each of the orbits in an atom or molecule tends to change its orientation precisely as an orbit in a magnetic molecule; but no gross magnetic effect can, on the theory advanced here, result, because the geometric sum of the individual magnetic moments of the particle is permanently zero.

§6. In the experiments performed here two modifications of the earlier method have been tried, in both of which the galvanometer was replaced by a fluxmeter. In one series of experiments the magnetic circuit was constituted almost wholly of iron. The iron cylinder under investigation rotated between the pole-faces of a large U-shaped electromagnet, with minute air gaps; and the similar coils on the two legs of the iron core, connected in series in the usual manner, were put in the series with the fluxmeter. This apparatus was very sensitive, but the large extraneous reactions were such as to mask or make impossible of interpretation such minute effects as were under investigation.

§7. The other series of experiments, which has given definite and conclusive results, will be described in considerable detail on account of the newness of the effect obtained.

In these experiments two nearly similar rods of steel[1] shafting A and B (Fig. 3) were mounted with their axes horizontal and approximately at right angles to the magnetic meridian, and two similar coils of insulated wire were mounted about their centers. These coils were connected in series with one another and with a fluxmeter, and were oppositely wound like the coils of an ordinary U-shaped electromagnet, so that any variations in the intensity of the earth's field, acting in the same way on both rods, might produce no effect on the fluxmeter. One of the rods, which will be called the compensator, as A, remained at rest, while the other, called the rotor, as B, was alternately rotated and brought to rest, the change of flux being determined by the fluxmeter.

The approximate dimensions of the rods A and B as they were used in nearly all of the work, are indicated in Fig. 3. The dimensions of the

[1] Steel, even when soft, has the advantage of freedom, or approximate freedom, from magnetic lag in weak fields, as shown long ago by Ewing (Roy. Soc. Proc., June 20, 1889), and confirmed by experiments on the steel rods used in this work.

two coils used in most of the work are also indicated in the figure. Each of these coils was wound on a brass bobbin with about 5,000 turns of No. 14 D.C.C. copper wire, the winding being made very regular on account of other investigations for which they were primarily designed.[1] A third coil, of somewhat different construction, was substituted for one of the coils in some of the experiments (see below, §23).

§8. In the first part of the work the rotor was mounted in brass bearing

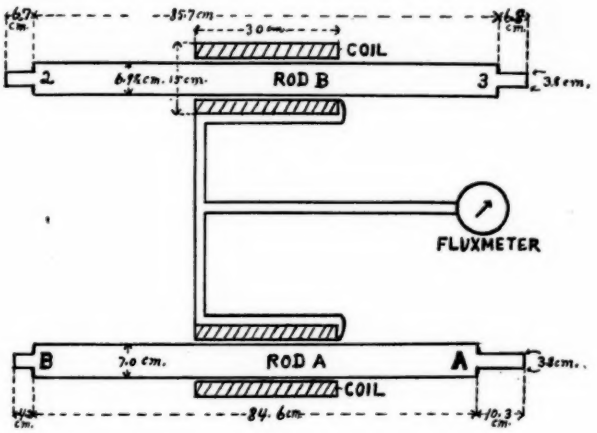

Fig. 3.

pieces inserted in bronze castings bolted to the cement floor of the laboratory, all parts being accurately fitted so that there was as little play as practicable. The rotor was driven by a brass rod 1.4 meters long and 1.1 cm. in diameter, itself direct-connected to a counter-shaft and pulley system driven by belt from an alternating current motor of the repulsion type.[2] The countershaft and pulley system was entirely of brass, bronze, and wood, and moved in brass castings bolted to the floor, with thin iron sleeves for bearing pieces. The motor's pulley was similar to that of the countershaft, except that it was mounted on an iron sleeve. Excessive vibration of the connecting rod was prevented by three properly spaced pieces of wood fastened to the floor and pierced by holes of suitable diameter for the passage of the rod. This method of driving, together with frequent oiling and the slow starting of the motor by the insertion of a choke-coil in series with its field coils, eliminated almost completely magnetic changes in the rotor due to vibration. The compensator and rotor were mounted 2.8 meters apart in positions symmetrical with respect to the motor.

[1] S. J. Barnett, "On electromagnetic induction and relative motion," PHYS. REV., 35, 1912, p. 323; "Some experiments on the magnetic field of two electromagnets in rotation," Phil. Mag., 26, 1913, p. 987; (London) Electrician, 74, 1914, p. 21.

[2] For the loan of this motor I am indebted to the department of electrical engineering.

§9. The fluxmeter was a Grassot instrument provided with a concave mirror with approximately 2 m. radius of curvature. It was mounted on a slate shelf cemented into the laboratory wall, and readings, with lamp and scale, were obtained to 0.1 mm. at a scale distance of approximately 2 m. Later on the scale distance was increased, usually to 8 m., a concave lens being added,[1] and all deflections have been reduced to this distance.

On account of the residual torsion of the suspension and the thermal electromotive forces in the circuit, the fluxmeter image usually drifts across the scale with greater or less rapidity, and it is necessary to compensate the drift by introducing suitable electromotive forces into the circuit. This was done by means of a potentiometer arrangement such as that commonly used in experiments on the Hall effect. As a result the drift was largely compensated, but ordinarily by no means as well as in the later experiments.

§10. In order to eliminate residual drift and other extraneous disturbances, readings were ordinarily taken in sets of eight, as follows: two with counter-clockwise or *negative* rotation (as seen from the west end of the rotor) four with clockwise or *positive* rotation, and two more with counter-clockwise or negative rotation, the order of the positive and negative rotations being sometimes reversed. The fluxmeter scale was read when the speed became steady, and again when the speed became so small that a minute quaver of the image became just visible, very slow rotation always producing a synchronous small vibration of the image. Mean deflections for the two directions were then obtained; and half the difference gave the change of flux produced by changing the speed from its maximum (which was the same in both directions) to a few revolutions per second.

To make the rotor run as smoothly as possible and to keep the bearings as free from deterioration as possible, the bearings were almost invariably oiled before taking each observation. Observations were made as the motor came to rest, instead of being made as it was brought to full speed, because the disturbances, both mechanical and electrical, were less for this procedure.

§11. The fluxmeter was standardized for changes of flux by introducing into its circuit, connected as in the rotation experiments, one coil of a

[1] The equivalent radius of curvature of this optical system depended greatly on the temperature, varying within the course of the work by more than 2 m. The optical system was otherwise poor, but the readings could be obtained with precision by making use of the narrow diffraction pattern produced with a carbon lamp glowing at extra voltage, or one of the new nitrogen filled tungsten galvanometer lamps made by the General Electric Company. For the opportunity of using one of these excellent lamps before they were placed on the market I am indebted to Mr. L. T. Robinson of this company.

low resistance mutual inductance standard, and making and breaking a circuit containing the other coil traversed by a current measured with a standard ammeter. To a precision of about one per cent. the fluxmeter gave 1 mm. deflection for 90 maxwells at the scale distance 8 meters.

Drift being eliminated, special experiments showed that the fluxmeter deflections for minute flux changes of the order of those occurring in the rotation experiments recorded here were independent of the time in which the changes occurred—as they are well known to be for large flux changes.

§12. To determine H_0, the intrinsic magnetic intensity of rotation per mm. of fluxmeter deflection at the scale distance 8 meters, the procedure was as follows:

Two equal wooden cylinders turned to the same diameter as that of rotor B were fastened coaxially to its ends, and the complete cylinder thus formed was wound (in three parts) on a lathe with a nearly uniform solenoid of insulated wire, 8 turns to the inch. The complete solenoid was about three times as long as the rotor occupying the central part. The solenoid was connected in series through a key with a constant battery of electromotive force 2.02 volts and an adjustable standard resistance box. With a resistance of 5,000 ohms in the battery circuit, and the fluxmeter circuit arranged as in the rotation experiments, the fluxmeter deflection was obtained repeatedly when the battery circuit was reversed in both directions. Many previous experiments had proved that with such minute magnetomotive forces or currents as were involved in these experiments and the other calibrating experiments described below, fluxmeter deflections, as would be expected, were strictly proportional to currents.

If h_0 denotes the solenoid's magnetic intensity per mm. deflection at 8 m., and D the mean fluxmeter deflection at 8.5 m., the scale distance at which these observations were made, we have from what precedes and the well-known formula for the magnetic intensity within a long uniform solenoid

$$h_0 = \frac{\frac{4\pi}{10} \times \frac{8}{2.54} \times \frac{2.02}{5{,}000}}{\frac{8.0}{8.5} \times \frac{D}{2}} \frac{\text{gauss}}{\text{mm.}} \qquad (10)$$

The experiments were made with two different distances between rotor and compensator, viz., 1.2 m. and 1.6 m. The values of D for the two sets did not differ by as much as one third per cent. The mean value of D was found to be 273 mm., at the scale distance 8.5 m.

The long solenoid was desirable in order to obtain as nearly uniform a calibrating intensity as practicable, the intrinsic intensity of rotation

being strictly uniform throughout the rotor. On account of the fact that the calibrating intensity acts on both the iron and the aether which permeates it, while the intrinsic intensity of rotation acts upon the iron only, it was desirable to eliminate the effect of the solenoid alone by making a separate experiment with no iron in the solenoid. Experiments of this kind showed that the deflection produced by the solenoid alone was 1.6 per cent. of that produced by the solenoid and core.

From this fact and equation (10) we get for H_0

$$H_0 = h_0(1 + 0.016) = 1.26 \times 10^{-5} \frac{\text{gauss}}{\text{mm.}} \qquad (11)^1$$

Approximate calibrations were also made with a similar solenoid wound on the rotor alone,[2] with distances 2.8 m., 1/3 m. and 1/4 m., between rotor and compensator. These gave for H_0, in the unit used above, 1.25×10^{-5}, 1.14×10^{-5}, and 1.07×10^{-5}, respectively.

§13. To determine B_0 and I_0, the flux density and intensity of magnetization per mm. deflection at 8 m. produced by the rotation H_0, a second experiment was necessary. A coil of 200 turns of insulated copper wire was wound over the central 10 cm. of the long solenoid and connected in circuit with the fluxmeter, the total resistance of the fluxmeter circuit being made equal to its former value by the addition of extra resistance from a box. The mean fluxmeter deflection was then obtained on reversal of a solenoid current just four times as great (1,250 ohms being now in the solenoid circuit) as that used in the first experiment. For the rotor-compensator distances 1.2 m. and 1.6 m. the same deflection (within 1/7 per cent.), 70.6 mm., was obtained at the scale distance 8.5 m.

The cross-section of rotor B was 37.8 cm.^{2}, and the fluxmeter constant was, as stated in §11, 90 maxwells per mm. at 8 m. Hence if δB denotes the change of flux-density in the first experiment, and $\Delta B = 4\delta B$, therefore, that in the second, we have for the actual change of core flux in the second experiment

$$\Delta B \times 37.8 \text{ cm.}^2 = \frac{70.6}{200} \times \frac{8.0}{8.5} \times 90 \text{ maxwells.}$$

The same change of core flux with the first arrangement of coils would have produced at the scale distance 8 m. the deflection $4 \times 273 \times 8.0/8.5$ mm. Hence, as this was the arrangement used in the rotation experiments, we have for the flux-density produced by rotation per mm. deflection at the scale distance 8 mm.,

[1] Calibrations made before and after the principal experiments, and with methods and standardized instruments varied, were all in close agreement.

[2] Elimination of the extra length of solenoid almost exactly compensated for the small effect produced by the solenoid with iron removed.

$$B_0 = \frac{1}{37.8} \times \frac{70.6}{200} \times \frac{1}{4 \times 273} \times 90 \left(\frac{\text{maxwells}}{\text{cm.}^2} \right) \text{per mm.} \qquad (12)$$

$$= 7.7 \times 10^{-4} \left(\frac{\text{maxwells}}{\text{cm.}^2} \right) \text{per mm.}$$

In connection with the approximate determinations of H_0 at the rotor-compensator distances 2.8 m., 1/3 m., and 1/4 m., mentioned in the last section, determinations of B_0 were also made giving the values 8.1×10^{-4}, 7.3×10^{-4}, and $6.9 \times 10^{-4} \left(\frac{\text{maxwells}}{\text{cm.}^2} \right)$ per mm., respectively.

The intensity of magnetization at the central section of the rotor per mm. deflection, which will be denoted by I_0, is approximately

$$I_0 = B_0/4\pi. \qquad (13)$$

§14. With the apparatus now described over twenty-five sets of observations were obtained. The rotations of one group were made with rotor A, most of them at about 50 r.p.s., two at about 18 r.p.s. Those of the other group were made with rotor B, some of them with the end designated as "end 2" east, the remainder with the rotor reversed, all at about 50 r.p.s.

Each of the groups gave a mean result (differential deflection) in the direction predicted by theory on the assumption that the revolving electrons are negative, the mean deflection (reduced to 8 m. scale distance) per unit speed being about 0.04 mm.; but the discrepancies were large, partly on account of imperfect compensation of the earth's intensity owing to the large distance between rotor and compensator, partly on account of non-uniformity and imperfect compensation of fluxmeter drift, and partly on account of mechanical vibration and other causes.

§15. As had proved to be the case in the Louisiana experiments, there was always superposed on the deflection which apparently corresponded to the effect under investigation a deflection independent of the direction of rotation. Moreover, in one set of observations made on rotor A to see whether either effect was altered by reversing the residual magnetism of the rotor, it was found (and the same thing had been found in the earlier experiments), that no alteration occurred (see further, §34). But reversing the ends of a rotor invariably reversed the absolute deflection.

§16. In seeking an explanation of these phenomena and a justification of the method of investigation a number of experiments were made. In the first place it had been assumed that an alternating flux through the fluxmeter circuit would not affect the scale reading provided the frequency exceeded a few cycles per second. An alternating flux was

always produced by the rotation of the rotor on account of lack of symmetry of its residual magnetization with reference to the coil, and by the stray flux from the alternating current motor.

To investigate this matter, one coil of a low resistance mutual inductance standard was placed in the fluxmeter circuit and the effect on the reading noticed when the other coil was traversed by an alternating current of such magnitude as to produce changes of flux through the other coil and fluxmeter of the same amount as occurred during the slow rotation of the rotor. Absolutely no effect was produced by making or breaking the alternator's circuit, the frequency being 40 cycles per second. Even changes of ten and one hundred times this amount, at frequencies of about 33 and 63 cycles per second, respectively, produced either no effect or (in the case of the latter) sometimes a slight effect on breaking the circuit. Neither was any change produced by driving the alternator from rest up to full speed, the circuits being permanently closed.

A related test was made with the motor itself. The motor, on being started, usually produced a quick throw of a number of mm., sometimes in one direction, sometimes in the other. This irregular effect quickly disappeared and did not reappear on stopping the motor. To test the effect of the motor on the permanent deflection, the fluxmeter was disconnected from the rotor and compensator coils and connected to another short thick coil provided with an iron core and placed near the motor. With this arrangement, and the motor driving the rotor as before to insure the expenditure of the same power as in the principal experiments, irregular throws, similar to those which occurred in these experiments, were produced on starting the motor, but the change of reading on the motor's coming to rest was either very slight or zero.

Further experiments on the fluxmeter are described below.

Suspicion that the effects might be due to a very slight longitudinal displacement of the rotor, or a very slight angular displacement of its axis, was easily shown by experiments to be groundless. All possibility of electric disturbances, which gave considerable trouble in a part of the work referred to in §6, was eliminated by earthing the fluxmeter circuit.

§17. It seemed possible that the second effect mentioned in §15, also the curious difference between the effects of reversing the rod and reversing the magnetization, and perhaps also with them even the first effect could be explained as consequences of the induction of electric currents in the rotor by its rotation in the earth's magnetic field. An effect of such currents upon the rotor coil must ordinarily exist even though it is wound with the turns almost precisely perpendicular to the rotor.

The change in sign of the mean deflection on reversal of the rotor pointed toward inhomogeneity; and it seemed possible to account for the identity of sign of the deflections for both directions of rotation by the shift of the stream lines of the induced currents with the reversal of this direction.

Figs. 4 and 5 illustrate an extreme case, in which, on account of symmetry, the fluxmeter deflection would be the *same*, in both direction and magnitude, for both directions of rotation of the rotor, A. In the first figure the stream-lines, the cross-section of the symmetrical one of which is indicated by the black dots, are displaced anti-clockwise, in the second clockwise, with reference to the earth's intensity, with the result that

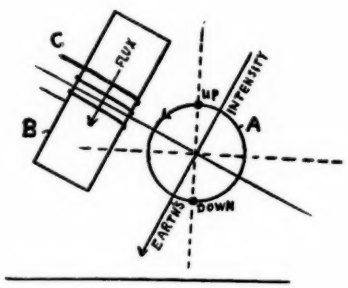

Fig. 4.

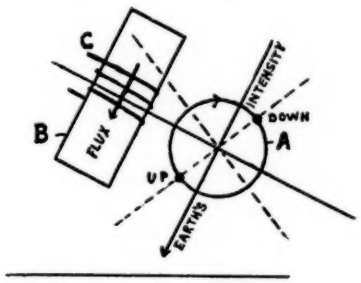

Fig. 5.

the change of flux through the test coil C is the same in both cases. It is easy to arrange a test coil near the rotor in such a way as to obtain almost any relation between the deflections for the two directions of rotation.

§18. An attempt was made to test this idea by altering as much as practicable the angle between the axis of the rotor coil and the axis of the rotor, although the free space between the rotor and bobbin was so small that only a small angular shift was possible. In the one complete and careful set of observations made, the mean deflections were altered considerably by the shift, but the sign of the deflections and the sign and magnitude of the differential deflections were not altered. In other, and rougher, experiments with different directions of the coil's axis, with one uncertain exception, no change of sign resulted. Moreover, later experiments, in which an irregularly wound coil was used, gave deflections in the same direction.

In another experiment the rotor coil was removed and mounted on a second compensating rod parallel to the first, the two being placed near and on opposite sides of the rotor. Rotating the rotor always produced a decided fluxmeter deflection.

§19. To get rid of the effects of induced currents in the rotor two methods were tried. First a rotor was constructed almost entirely of thin discs of iron, but its rigidity was insufficient. Resort was therefore had to the second alternative, viz., the compensation of the earth's intensity in the region occupied by the rotor by means of a large coil or cage of wire traversed by an electric current.

The framework of this cage was accurately constructed of brass and paraffined wood. The coil was about 1.5 m. in length (horizontal and approximately parallel to the rotor), $\frac{1}{4}$ m. in width (approximately normal to the earth's intensity), and $\frac{1}{7}$ m. in depth (approximately parallel to the earth's intensity). It was carefully wound of copper wire, uniformly except at the ends, where provision was made for the admission of the driving rod and rotor, and was mounted on the floor with its center approximately at the center of the rotor. A coil of these dimensions cannot, of course, produce a strictly uniform field in the region occupied by the rotor, and a much larger coil would have been made except for the expense involved in its construction and in properly mounting the rotating apparatus at a considerable height above the floor. Nevertheless it was capable of giving important information, and with it the first conclusive results were obtained.

§20. A few sets of observations on rotor *B*, with both end *2* and end *3* east, taken with the earth's field approximately annulled in this manner, gave results practically the same as those already described.

§21. In spite of the tests already described, suspicion still attached to the electric motor; and for a greater degree of certainty it was now replaced by a small air turbine made by adding to the countershaft and wooden pulley already described six uniformly spaced radial brass vanes. Compressed air was admitted to the vanes from four brass nozzles, two, symmetrically placed about the axis of the turbine, being open for each direction of rotation. The bronze shaft of the turbine was directly connected to the rotor by a brass rod 1.1 cm. in diameter and 0.3 m. long. For the final experiments the length was somewhat increased.[1]

Uncertainty as to the steadiness of the air pressure and speed made it necessary to add an instantaneous speed counter. For this purpose a very small separately excited dynamo, with a voltmeter across the armature terminals, was used. The dynamo was driven by a 24 ft. belt running on a pulley on the shaft of the turbine and the combination was properly calibrated, speeds in revolutions per second being obtained

[1] The rotation of the bronze countershaft, turbine wheel, and connecting mechanism, while the rotor was clamped, produced no effect on the fluxmeter although the speed was greater than the maximum speed of the rotor in the principal experiments.

by multiplying the voltmeter reading by 0.566.[1] The speeds were sufficiently near to uniformity, the means for the clockwise and counter-clockwise rotations in a set not differing usually by as much as one r.p.s., and the individual speeds often differing by much less than this.

§22. The observations with this apparatus were made in much the same manner as those already described, except that the fluxmeter reading for zero speed was obtained for a definite position angle of the rotor, instead of the reading for low speed. The turbine wheel was numbered at equal intervals around the periphery, and the zero reading was taken with a chosen one of these numbers uppermost, always the same throughout a set. The absolute deflection depends upon the number chosen, but the differential deflection does not, as was early found by direct experiment. This will be explained in §30.

§23. In this manner a number of sets of observations were obtained with both rotors and under various circumstances. These observations are given in Table I. For some of them the distance between rotor and compensator was made as small as ¼ or ⅓ meter, in order to compensate more readily the variations of the earth's intensity; but at such small distances apart the inductive action between the two rods is sufficient to modify the mean deflection greatly as is evident from the effect of reversing the compensator; though the effect on the differential deflection is very much less. In the last column but one, E signifies that the rotor was in the uncompensated field of the earth; $2E$ that the earth's field was approximately doubled in intensity by means of a current in the compensating coil; $-E$ that the earth's intensity was approximately reversed by a compensating current, and O that the earth's intensity was approximately compensated by the current in the cage.

It is to be remarked that in the case of every group (and the same is true of every set), however different the circumstances, the differential deflection has the same sign, and the positive sign, which, for the given arrangement of apparatus, means an effect in the direction predicted by theory on the assumption that the revolving electrons are negative.

Sets 13–18 were obtained with a rotor coil different from the two already described. It contained the same amount of the same kind of wire and had approximately the same dimensions and same constant in the fluxmeter circuit, but was not wound so regularly; and it was wound on a bobbin made of wood and fiber. This substitution was made in order to annul any possible effect of induced currents in the bobbin due to the lack of symmetry of the rotor's magnetization about

[1] After all the rotations in this investigation were completed this calibration was checked and found to have remained unaltered.

TABLE I.

Deflections and Speeds.

Earlier Observations and Results.

Number of Group.	Number of Sets.	Rotor Used.	End East.	Distance Between Rotor and Compensator.	Mean Speed.	Mean Deflection for Negative Rotation (−).	Mean Deflection for Positive Rotation (+).	Differential Deflection (−) − (+).	Intensity of Field.	Remarks.
				m.	r.p.s.	mm.	mm.	mm.		
1	5	*B*	2	¼	42	+ 9.3	+ 7.1	+2.2(1.9)[1]±0.9	0	
2	4	*B*	3	¼	45	+ 3.0	− 1.4	+4.4(3.7)[1]±1.1	0	
3	2	*A*	*B*	⅓	42	−11.0	−13.1	+2.1(1.9)[1]±0.2	0	
4	2	*B*	2	⅓	23	− 0.6	− 1.7	+1.1(1.0)[1]±0.1	0	Compensator reversed in Group 5. Mean reduced diff. defl. = +0.95; Mean speed = 26½
5	1	*B*	2	⅓	30	+ 2.5	+ 1.5	+1.0(0.9)[1]	0	
6	1	*B*	3	¼	47	− 1.3	− 4.4	+3.1(2.6)[1]	2*E*	
7	1	*B*	3	¼	45	− 0.5	− 4.9	+4.4(3.7)[1]	*E*	
8	1	*B*	3	¼	45	+ 4.6	+ 3.1	+1.5(1.3)[1]	− *E*	
9	1	*B*	2	¼	45	+15.4	+10.5	+4.9(4.2)[1]	*E*	
10	1	*B*	2	¼	45	+11.6	+ 8.8	+2.8(2.4)[1]	− *E*	
11	1	*B*	2	1	42	+ 4.4	+ 1.8	+2.6	0	End *B* of compensator east.
12	1	*B*	2	1	39	+ 5.1	+ 2.8	+2.3	0	End *A* of compensator east.
13	1	*B*	2	1	45	+ 4.9	+ 1.0	+3.9	*E*	Rotor coil on insulating bobbin.
14	2	*B*	2	1	46	+ 7.0	+ 4.3	+2.6±0.5	0	Rotor coil on insulating bobbin.
15	2	*B*	2	1	32	+ 3.6	+ 1.9	+1.8±0.5	0	Same, also rotor in wooden bearings.
16	1	*B*	2	1	33	+ 4.0	+ 1.8	+2.2	0	Same, also cage circuits broken.
17	2	*B*	2	1	39	+ 5.9	+ 3.3	+2.6±0.2	0	Same, also iron bolts removed.
18	1	*B*	2	1	40	+ 8.0	+ 3.2	+4.8	*E*	" " " " "

[1] The quantities in parentheses are the deflections reduced to the sensibility for the case in which rotor and compensator were 1 meter or more apart.

its axis. For a similar reason wooden bearings were then substituted for the metal bearings, and the brass circuits of the cage were cut. It was finally discovered that four small iron bolts had been accidentally left embedded in the cement floor, two near each end of the rotor (whose axis was about 13 cm. from the floor). These were removed. Observations made after each of these changes showed that they were without effect.

§24. The relation between the differential deflection and the speed was plotted on cross-section paper for groups 4–5, 11–12, and 14–17. As the table shows, 11–12, and 14–17 are the only groups for which the earth's intensity was annulled and the rotor-compensator distance as great as 1 m.; and groups 4–5 are the only ones with smaller distance and with earth's field annulled and compensator in both positions. Within the limits of the experimental error the deflection was proportional to speed. The mean of 4 and 5 was taken in order to eliminate as far as was possible the effect of the compensator. A straight line was drawn from the origin with slope obtained by dividing the sum of all the differential deflections by the sum of all the speeds, each group being counted as many times as it contained sets, except in the case of 4–5, which was considered equivalent to only 2 sets. The slope of this line was the weighted mean differential deflection per unit speed and equals 0.057 mm. per revolution per second.

§25. After the completion of the work thus far described it was decided to repeat the rotations in a region in which the earth's intensity was still more completely annulled.[1]

For this purpose it was necessary to mount the rotor and turbine about half a meter above the floor, and suitable piers were constructed of concrete. The two piers which supported the rotor were necessarily narrow (though much longer than broad) and inclined to the horizontal at about the angle of dip, and were therefore reinforced with brass rods cemented with them into the floor. The iron bearing sleeves of the turbine were replaced by vulcanized fiber, the iron pipe carrying compressed air to the turbine tubes was replaced by brass, and the earth's magnetic field in the region to be occupied by the rotor was carefully examined. In passing from one end of the rotor to the other the declination varied by about one degree, and the dip was uniform to at least one tenth degree. The rotor was mounted with its axis horizontal and perpendicular to the mean meridian. The original brass bearings in bronze castings were used, as they had been shown to be without injurious

[1] The desirability of this course was realized from the first and was also mentioned by Dr. Rosa at the Philadelphia meeting of the Physical Society, Dec., 1914.

effect. The box to hold the compensating coil was rectangular in cross-section and about ¼ meter wide, like that used before, but was about 2 meters long and 1 meter deep. The framework was strongly and accurately built, chiefly of 2 in. × 4 in. and 2 in. × 2 in. well seasoned yellow pine. Over the two larger sides of this frame were screwed, in small sections, accurately planed boards of well seasoned white pine provided with accurately cut parallel and equally spaced uniform grooves to hold the wire. Boards of the same kind, except that they were not grooved, completed the wooden surface over which the coil was wound. All materials were non-magnetic, and all the woodwork was coated with paraffine to prevent warping. The coil was wound with 151 turns of No. 20 copper wire, each turn being all in one plane, except for the short end portions, which went over the ungrooved surfaces and were somewhat irregular. The passage from one turn to the next was made by a sharp turn at one corner of the frame, and the return wire was carried close to this corner. The wires were fastened to the bottoms of the grooves and held in place by paraffine. The width of the box was about 26.8 cm. (26.9 at the middle, 26.7 at the ends), the length about 197.9 cm., and the coil was 95.9 cm. long.

§26. To test the degree of uniformity of the part of the magnetic field produced by the electric current in this coil in the region to be occupied by the rotor, the procedure was as follows: A test coil about 6 cm. long and 6 cm. in diameter, connected in circuit with the fluxmeter, was so mounted on a wooden support that it could be set with its axis parallel to that of the compensating coil and its center in the central horizontal line of 148 consecutive turns of the cage which alone were used in this part of the experiment. The test coil was moved along by measured steps from a point near one end of the cage to a point near the other, and in each position the fluxmeter deflections were obtained twice for reversals of the current in the cage; and the procedure was repeated with the coil moving in the other direction. The current remained constant within about one fourth per cent. during the whole process.

The mean fluxmeter deflections were then plotted on cross-section paper as a function of the position of the test coil. In this way it was found that the intensity produced by the cage did not vary by quite as much as 1 per cent. of its mean throughout the region occupied by the main cylindrical part of the rotor, and that it varied only 1.4 per cent. in passing from one end of the rotor to the other end, becoming of course greater as the ends were approached. Over the greater portion of the length of the rotor the intensity did not vary by more than one third or one quarter per cent. The variation would have been slightly

less had the 151 turns been used as they were in the rotation experiments.

§27. The compensating cage (with 151 turns) was now mounted over the rotor and its coil, the centers of all three being made nearly coincident. The cage was adjusted with the longer edges parallel to the rotor and its axis parallel to the earth's intensity, and was clamped to the floor in this position. The cage current necessary to compensate the earth's intensity was then determined. With this end in view, a bunched longitudinal coil of thin insulated copper wire had been wound over the central four tenths of the rotor, two sides of the coil being diametrically opposite to one another, and the other two being semicircular and on the same side of the rotor. This coil was connected in circuit with the fluxmeter and set with its axis parallel to the axis of the cage and to the earth's intensity. Turning the rotor through 180° produced a deflection of 17.3 cm. with no current in the cage. With a current of 335 milliamperes in the proper direction in the cage, the deflection on reversal was not 0.1 mm. This current, therefore, compensated the earth's intensity through the central region of the rotor within a minute fraction of 1 per cent., and somewhat less completely throughout the rest of the rotor, as appears from §26.

After the determination of the compensating current the longitudinal test coil was removed from the rotor, and the fluxmeter and rotor and compensator coils connected for the main experiments.

§28. In studying the earth's intensity in the region to be occupied by the rotor it was found that rotation of the fluxmeter through 360° about the vertical in its former position caused the declination to vary through a range of about half a degree. For this reason and because it was desired to have a greater range of scale distances available, the fluxmeter was removed to a very large room adjacent to that in which the rotations occurred, and was again mounted on a slate shelf cemented into the wall. In the later and more precise of the experiments already described the compensation of the fluxmeter drift had been greatly improved by increasing the thermal insulation of many of the junctions of the circuit. For the final work this insulation was still further improved. By taking this precaution, and by working at night, when air currents were reduced, the drift was made so regular (when it did not vanish, as it usually did not) that its compensation was not difficult to make practically perfect. On the other hand the building, though sometimes completely free from vibration, was often disturbed. This however, is by no means so serious a trouble with a compensated fluxmeter as with an instrument whose readings are not permanent; and observations were obtained which were superior to most of those given above.

TABLE II.

Deflections and Speeds. Final Observations and Results.

1915 Date.	Number of Set.	Rotor End *E*.	Compensator End *E*.	Deflection for Negative Rotation (−).	Deflection for Positive Rotation (+).	Differential Deflection (−) − (+).	Zero Correction.	Speed for (−).	Speed for (+).	Mean Speed.	Field Intensity.	Distance from Rotor to Compensator.
				mm.	mm.	mm.	mm.	volts	volts	r.p.s.		m.
March 5	1	2	*B*	−4.7	−3.2	−1.5	−0.6	61	61	34.5	0	1.2
" 5	2	2	*A*	−5.0	−3.3	−1.7	−0.6	65	65	36.8	"	"
" 12	3	2	*A*	−4.9	−3.6	−1.3	−0.6	60	64	35.1	"	"
				(−5.4)[1]		(−1.8)[1]		(64)[1]		36.2		
" 12	4	2	*A*	−5.0	−3.4	−1.6	−0.6	60	60	34.0	"	"
" 13	5	2	*A*	−4.8	−2.7	−2.1	−0.6	64	59	34.8	"	"
				(−4.2)[1]		(−1.5)[1]		(59)[1]		33.4		
" 13	6	2	*A*	−3.2	−1.3	−1.9		63	63	35.7	− *E*	"
" 13	7	2	*A*	−7.2	−5.7	−1.5		66	61	35.9	+ *E*	"
" 16	8	2	*A*	−3.4	−2.1	−1.3	+0.6	61	65	35.7	0	"
				(−4.0)[1]		(−1.8)[1]		(65)[1]		36.8		
" 16	9	2	*A*	−4.3	−2.7	−1.6	+0.6	69	69	39.1	"	"
" 16	10	2	*A*	−1.4	−0.4	−1.0	+0.6	42	41	23.5	"	"
" 16	11	2	*A*	−1.3	−0.5	−0.8	+0.6	40	40	22.6	"	"
" 18	12	3	*A*	+2.5	+4.4	−1.9		70	68	39.1	"	"
				(+2.3)[1]		(−2.1)[1]		(68)[1]		38.5		
" 18	13	3	*A*	+2.4	+4.6	−2.2		70	69	39.3	"	"
				(+2.3)[1]		(−2.3)[1]		(69)[1]		39.1		
" 18	14	3	*A*	+1.9	+4.0	−2.1		70	67	38.8	"	"
				(+1.6)[1]		(−2.4)[1]		(67)[1]		37.9		
" 22	15	2	*A*	−3.9	−1.4	−2.5	+1.2	70	68	39.1	"	1.6
				(−3.6)[1]		(−2.2)[1]		(68)[1]		38.5		

[1] The quantities in parentheses are the deflections for negative rotations, the differential deflections, and the speeds for negative rotations corrected to the speeds of the positive rotations.

TABLE II.—*Continued.*

1915 Date.	Number of Set.	Rotor End *E*.	Compensator End *E*.	Deflection for Negative Rotation (−).	Deflection for Positive Rotation (+).	Differential Deflection (−) − (+).	Zero Correction	Speed for (−).	Speed for (+).	Mean Speed.	Field Intensity.	Distance from Rotor to Compensator.
				mm.	mm.	mm.	mm.	volts	volts	r.p.s.		m.
March 22	16	2	*A*	−3.9	−1.3	−2.6	+1.2	71	68	39.3	"	"
				(−3.5)[1]		(−2.2)[1]		(68)[1]		38.5		
" 22	17	2	*B*	−3.8	−1.7	−2.1	+1.2	71	71	40.2	"	"
" 22	18	2	*B*	−3.7	−1.5	−2.2	+1.2	72	72	40.8	"	"
" 22	19	2	*A*	−3.7	−1.6	−2.1	+1.2	71	71	40.2	"	"
" 23	20	2	*B*	−0.7	+0.4	−1.1	+1.2	37	35	20.4	"	1.3
				(−0.5)[1]		(−1.0)[1]		(35)[1]		19.8		
" 23	21	2	*B*	−0.8	+0.2	−1.0	+1.2	41	40	22.9	"	"
" 23	22	2	*B*	−0.5	+0.3	−0.8	+1.2	38	38	21.5	"	"
" 23	23	2	*B*	−0.5	+0.4	−0.9	+1.2	37	37	20.9	"	"
" 23	24	2	*B*	−0.6	+0.5	−1.1	+1.2	36	36	20.4	"	"

[1] The quantities in parentheses are the deflections for negative rotations, the differential deflections, and the speeds for negative rotations corrected to the speeds of the positive rotations.

§29. The results of 24 sets of observations made under the improved conditions are given in Table II. Two sets were rejected because of great difference in the speeds for right handed and left handed rotations; and one set, in the middle of which an accident happened to the turbine, was rejected because of great speed differences and also because it was obtained on an *afternoon*, when the fluxmeter could not be well compensated either for thermal effects in its own circuit or, apparently, for variations of the earth's field (the distance from rotor to compensator on this occasion was about 2.2 meters). The retention of the rejected observations, which were quite consistent with those retained, would not have affected in any way the conclusions to be drawn from the experiments.

For these observations the fluxmeter circuit happened to be connected up in such a way that a negative value of the differential deflection corresponded to an effect in the direction predicted by theory on the assumption that the electrons in revolution are negative. The negative sign here is thus consistent with the positive sign in Table I.

In most of the sets the speeds for the two directions of rotations were nearly the same. When they differed by more than one volt (or about half a revolution per second), except in the case of those observations made in a magnetic field, the deflections for the negative rotation and the corresponding differential deflections, have been reduced to the speed of the positive rotation by the method explained in §32. The reduced

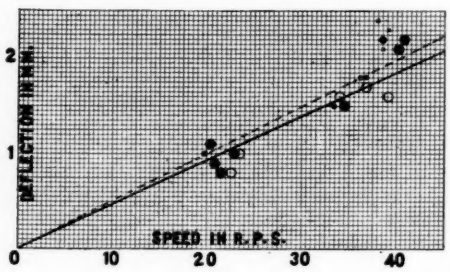

Fig. 6.

Differential Deflection and Speed.

values, along with the common reduced speeds are given in parentheses.

All the deflections were obtained at 8 meters scale distance except those of March 22, which were obtained at 8.7 meters. In the table the deflections for these observations are reduced to 8 meters.

The differential deflection is plotted as a function of the speed in Fig. 6, sets 6 and 7, which were obtained with the rotor in the magnetic field of the earth, or that field reversed, being omitted. The large circles

designate the observations which needed no reduction for speed differences; the smaller circles and crosses, those for which reductions were made. The circles designate observations made with end *2* of the rotor east; the white circles corresponding to those made with the compensator end *A* east, the black circles to those made with compensator end *B* east; the crosses designate observations made with the rotor end *3* east and compensator end *A* east.[1]

The continuous straight line in the figure is drawn with a slope obtained by dividing the sum of all the differential deflections for the observations which needed no reductions by the sum of all the corresponding speeds, and thus gives the weighted mean deflection per unit speed, on the assumption of proportionality between them. This quantity is 0.046 mm./r.p.s.

The broken straight line is drawn in the same manner for all the observations made with the earth's intensity compensated, reductions having been made when necessary, as indicated above. From this line, the mean deflection per unit speed is 0.050 mm./r.p.s.

It is clear from the figure that the assumption made above is justified: the deflection is proportional to the speed within the limits of the experimental error. It is also clear that within these limits this deflection is independent of the orientation of the rotor or compensator or the distance between them, this distance being always at least as great as 1.2 meter. The average departure of the differential deflection for a single set from the ordinate of the (broken) straight line corresponding to the speed is 12 per cent.

§30. In the observations just described the readings for zero speed of the rotor throughout each set were, as before, obtained with the rotor set at a definite position angle. The absolute deflection depended upon this setting; but, as has already been said, it was early proved that the differential deflection did not. It was nevertheless considered important to investigate this matter more fully.

For this purpose a permanent bar magnet was mounted so as to be free to rotate in a horizontal plane about a vertical axis passing through its center, and arrangements were made to rotate it at the speed of about 26 revolutions per second by a small alternating current motor at a distance from the magnet and the coils of the fluxmeter circuit. Near the magnet was clamped a small low resistance coil of insulated copper wire connected in the fluxmeter circuit arranged (except for this addi-

[1] In the original drawing, in the upper right-hand corner, two small white circles on the same center are shown in place of the smallest of the group of three black circles shown in the reproduction. Also, the second spot in the group appears in the original as a large white circle surrounding a large black circle with the same center.

tion) as in the principal experiments. Another coil, with ten turns of insulated wire, was wound on the compensator close to the coil, and was connected through a key and an adjustable high resistance box with the terminals of a storage battery.

By means of this coil and battery circuit any desired change of flux could be produced in the fluxmeter circuit; and by means of the rotating magnet and the adjacent coil in the fluxmeter circuit the effect of the iron rotor in sending an alternating current through this circuit, and in producing a motion of the lamp's image up and down on the scale for very slow rotations, could be closely imitated.

The magnet was rotated slowly and the extreme scale readings A and B $(B > A)$ noted. On the magnet's being rapidly rotated the scale reading C always became the mean of the two readings, viz.,

$$\tfrac{1}{2}(A + B).$$

The fluxmeter deflection D produced by closing the battery circuit was determined while the magnet was at rest. Then the circuit was closed, and the magnet driven up to full speed, and the scale reading E determined; then the magnet was brought to rest and set in the position giving one of the extreme scale readings A. It was always found that

$$D = (E - A) + \tfrac{1}{2}(A - B).$$

These observations were made with two quite different values of $A - B$, and two quite different values of D; some of the experiments were made with the battery circuit closed before the rotation of the magnet was started, others with this order reversed. But the result was always the same.

We have thus conclusive proof that, as would be expected, the fluxmeter deflection, when reckoned from the proper zero C, gives correctly the flux changes. If one of the extreme points A or B has been taken as the zero, the true change of flux is given by the above equation.

For most of the observations given above the rotor was set for the zero reading at or close to one of the positions giving the readings A and B, and $A - B$ was determined. The corrections to the true zero C could then be readily made. These corrections are given, for the sets for which they are available, in Table II. This correction is of course of no consequence for the differential deflection, but it affects the absolute deflection greatly.

§31. When the means of the absolute deflections for positive and negative rotations are corrected to the true zero in accordance with the last article and compared with the speeds, a very simple relation is found to exist between them. In Table III. the observations are divided

TABLE III.

Mean Deflections and Squares of Speeds.

Numbers of Sets.	Sum of Deflections (Observed).	Sum of Deflections (Reduced).	Mean Deflection (Observed).	Mean Deflection (Reduced).	Sum of Speeds Squared (Observed).	Sum of Speeds Squared (Reduced).	Mean Square of Speed (Observed).	Mean Square of Speed (Reduced).	Mean Deflection per Unit Speed Squared (Observed).	Mean Deflection per Unit Speed Squared (Reduced).
1–5, 8–9, 15–19	43.8 mm.	43.7 mm.	3.65 mm.	3.64 mm.	16,920 (r.p.s.)^2	16,870 (r.p.s.)^2	1,410 (r.p.s.)^2	1,406 (r.p.s.)^2		
10–11, 20–24	9.4	9.3	1.34	1.33	3,320	3,290	474	471		
1–5, 8–9, 10–11, 15–19, 20–24	53.2	53.0			20,240	20,160			$\frac{\text{mm.}}{(\text{r.p.s.})^2}$ 0.00263	$\frac{\text{mm.}}{(\text{r.p.s.})^2}$ 0.00263

into two groups, those at lower speeds and those at higher speeds, and for each group are given the sum of the observed deflections and the mean deflections (both corrected to the true zero), the sum of the squared speeds, and the mean of the squared speeds; while for both groups together there is given the quotient of the sum of the deflections by the sum of the squared speeds, or the mean deflection per unit speed squared, all quantities being given both as observed and also as reduced to the same speed for both directions of rotation.

In Fig. 7 the mean squared deflections for the two groups are plotted as a function of the mean squared speed. The straight line is drawn

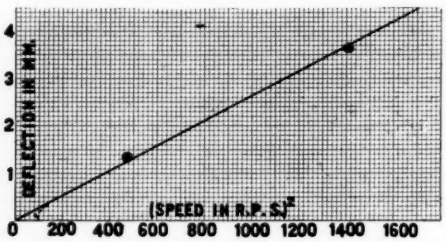

Fig. 7.

with the slope given by the last two columns in the table, which is identical for both observed and reduced quantities, as are the mean deflections within the experimental error. The figure shows that within the limits of the experimental error the absolute deflection is proportional to the square of the speed.

§32. The method of reducing a deflection observed at one speed to the deflection which would have been produced at another, and thereby correcting the differential deflections obtained when the speed for left hand rotations did not equal that for right hand rotations can now be readily explained. If we denote by D the deflection (from the true zero) obtained for a speed S, and by D' the deflection which would have been obtained for a speed S' near S, we have

$$D = aS + bS^2$$

and

$$D' = aS' + bS'^2,$$

where a and b are constants known very approximately from the observations which needed no correction. The correction, or small quantity which must be added to D, is

$$D' - D = a(S' - S) + b(S'^2 - S^2).$$

From Fig. 6 we get

$$a = \pm \tfrac{1}{2} \times 0.046 \frac{\text{mm.}}{\text{r.p.s.}} \quad \text{or} \quad \pm 0.013 \frac{\text{mm.}}{\text{volt}},$$

and from Fig. 7 we get

$$b = \pm 0.00263 \frac{\text{mm.}}{(\text{r.p.s.})^2} \quad \text{or} \quad \pm 0.00084 \frac{\text{mm.}}{(\text{volt})^2}.$$

As an example, what precedes may be applied to set 5, Table II., to reduce the deflection for left handed or negative rotations at speed 64 volts to the speed of 59 volts. In this case we have

$$D' - D = \{ - 0.013(59-64) - 0.00084(59^2-64^2) \} \text{ mm.} = + 0.6 \text{ mm.}$$

By adding this quantity to the observed deflection $-$ 4.8 mm., and subtracting $-$ 2.7 mm., we obtain the differential deflection $-$ 1.5 mm. for the speed 59 volts.

In this way all the differential deflections given in parentheses in Table II. were obtained. The total correction being always small, it is not important that the values of a and b should be accurately known. By using *all* the differential deflections for zero intensity of field, reduced in this manner when necessary, we obtain a more nearly correct value of a, viz.: $a = \pm 0.025$ mm./r.p.s., corresponding to the slope of the broken line in Fig. 6.

§33. The intensity of the magnetic field in which the rotor moved having been reduced in the final experiments to a minute portion of the earth's intensity, it is clear that that part of the deflection which is proportional to the square of the speed, or the mean deflection, is not produced by induced currents in the rotor.

The proportionality of the deflection to the square of the speed, and the reversal of the deflection with the reversal of the rotor, strongly suggest that the effect is due to the radial expansion (and accompanying longitudinal contraction) of the rotor produced by its rotation.

It has long been known that the longitudinal magnetic flux through an iron rod in or out of a longitudinal field is altered by the application or removal of longitudinal tension;[1] and that the longitudinal flux through the walls of an iron tube in a longitudinal field is altered by radial expansion or contraction produced by the application of hydraulic pressure from within or its removal.[2] The application and removal of longitudinal tension produce opposite effects; likewise radial expansion and contraction produce opposite effects. But longitudinal contraction and radial expansion produce effects of the same sign.[2]

Most of the experiments on this subject have been made on iron wires or cylinders in longitudinal fields, only a few having been made on iron rods with residual (or permanent) magnetization. For permanently

[1] Ch. Matteucci, Ann. de chim. et de phys. (3), 153, 1858, p. 416; E. Villari, Pogg. Ann., 126, 1865, p. 87.

[2] W. Thomson, Roy. Soc. Phil. Trans., 170, 1880, p. 64.

magnetized iron rods Matteucci[1] and Villari[1] found that elongation produced an increase of flux in the case of soft iron, but a decrease in the case of hard iron or steel. All gradations were found to exist between the two extremes.

In all of the very numerous experiments made with the two different rotors A and B used in this work, with the exception of one set mentioned below (§34), the mean deflection produced by rotation was in the direction indicating that the longitudinal flux through the rotor was *increased*. To test the above explanation of the effect it was necessary, on account of the dependence of the known mechanical effect upon the nature of the rod and perhaps upon the intensity of magnetization, to make direct experiments on the effect of *longitudinal compression* (and its removal) on the magnetization of the two rotors. The residual flux density in the central section of rotor A was about 7 maxwells per cm.^{2}; that in rotor B about 4 maxwells per cm.^{2}

Each rotor in turn was therefore placed in a strong wooden frame constructed for the purpose, and longitudinal compression was applied by means of a brass screw working in a brass nut and operated with a brass wrench. With the fluxmeter and coils arranged as in the principal experiments and the axes of the rods in the magnetic equator, it was repeatedly verified that *compression increased* the magnetic flux through the rod, and that removal of the compression brought the flux back to its initial value. In the case of rod A, 4 or 5 cm. deflection was produced by a half-turn of the screw; in the case of rod B the deflection produced by a half-turn was 3 or 4 cm. These experiments thus confirm the explanation given above of the mean deflection produced by rotation.

A part of the effect may be due to possible unsymmetrical distribution of the rotor's mass about its axis and consequent centrifugal distortion. This also would give an effect proportional to the square of the speed. Experiments in which rotor B, with ends fixed, was subjected to a lateral force of about 150 lbs. near one end of its coil gave a deflection of a few mm. in the direction indicating increase of magnetic flux when the force was applied and decrease when it was removed.

§34. The apparently exceptional set of observations referred to above, in which the mean deflection appeared to indicate a diminution of magnetization, was made over a year ago with rotor A. When the magnetization of this rotor was reversed, as it was thought, the deflection remained exactly the same. The reversing magnetic intensity applied, however, was small; and in view of the well-known van Waltenhofen phenomenon it is quite possible that true reversal did not occur. The

[1] Ch. Matteucci, Ann. de chim. et de phys. (3), 153, 1858, p. 416; E. Villari, Pogg. Ann., 126, 1865, p. 87.

direction of the magnetization was tested, but only by noting the direction of the deflection when the rod was smartly tapped, a test which has not always been found conclusive. A conclusive test is always to determine the direction of the fluxmeter deflection on the removal of the rod from its coil. This same result was obtained in the preliminary observations made in Louisiana, with a rod about half as long. To this, however, but little weight can be attached.

Anomalous effects of perhaps the same kind were noted by Villari (l. c.) in the case of longitudinal tension. He found that it was possible by magnetizing a steel rod with a current in one direction, and then with a weaker current in the opposite direction, to make it behave like iron—so that elongation increased its magnetization instead of decreasing it.

The exceptional cases mentioned therefore do not invalidate in any way the explanation of the mean deflection presented in the last article. It is of course not credible that reversal of the magnetization throughout a rod should have a different effect on the deflection produced by rotation from that of simply reversing the rod, an extraneous field being excluded as in the final experiments described here.

§35. In connection with the Louisiana experiments one of my friends suggested that the effects might be due to torsion, but was inclined to withdraw his suggestion when he found that the rotating rod was about 7 cm. in diameter.

In the case of a permanently magnetized rod of soft iron, according to the experiments of G. Wiedemann,[1] opposite small twists produce equal and opposite reversible effects on the magnetization.[2] It follows from this that the differential deflection would be increased by torsion for one orientation of the rotor, and decreased by the same amount for the opposite orientation. The experiments show, as would be expected, no difference produced by such a reversal.

Some direct experiments on rotor *B* confirmed Wiedemann's statements. For a torque of about 20 lb.-ft. applied to one end of the rotor while the other end was clamped, a mean deflection of about 2 mm. was produced. Experiments were made with both orientations of the rotor and for both directions of twist.[3] When the rotor was running at a speed of 38 r.p.s. in the principal experiments, the torque applied to one end of the rotor by the driving rod was only 1.1 lb.-ft., and the other end of the rod was of course free, except for the frictional torque. The effect

[1] G. Wiedemann, Elektricitaet, III., p. 676.

[2] The effects are similar in the case of steel except that hysteresis also occurs.

[3] Wiedemann does not give the direction of the effect. My experiments gave a diminution of flux when the positive end of the rod was twisted clockwise relative to the negative end to one looking along the rod from the negative to the positive end.

of torsion on any of the deflections observed in the experiments on rotation must thus have been far less than the smallest readable quantity.

§36. As a result of all that precedes we are forced to the conclusion that the rotation of a rod of steel[1] produces therein an intrinsic magnetic intensity proportional to the angular velocity and acting in a direction opposite to the intensity which would be produced by an electric current flowing around the rod in the direction of rotation, as required by the theory proposed in §§1 and 5.

To obtain the intrinsic magnetic intensity per unit speed it is now necessary only to multiply half the mean differential deflection per unit speed, given in §29, by the intrinsic intensity per unit deflection, H_0, given in §12. In this way we obtain

$$\frac{H}{n} = -\tfrac{1}{2} \times 0.050 \frac{\text{mm.}}{\text{r.p.s.}} \times 1.26 \times 10^{-5} \frac{\text{gauss}}{\text{mm.}} = -3.1_5 \times 10^{-7} \frac{\text{gauss}}{\text{r.p.s.}}. \quad (13)$$

Similarly, from §§29 and 13, we obtain for the magnetic flux-density per unit speed produced by the intrinsic intensity of rotation in the central part of the rotor, the quantity

$$-\tfrac{1}{2} \times 0.050 \frac{\text{mm.}}{\text{r.p.s.}} \times 7.7 \times 10^{-4} \left(\frac{\text{maxwells}}{\text{cm.}^2}\right) \text{per mm.}$$

$$= -1.9 \times 10^{-5} \left(\frac{\text{maxwells}}{\text{cm.}^2}\right) \text{per r.p.s.}$$

This quantity divided by 4π gives an approximate value of the intensity of magnetization at the center of the rotor per unit speed, viz.,

$$-1.5 \times 10^{-6} \text{ c.g.s. unit per r.p.s.}$$

The magnitude of H/n given in equation (13) as a result of the last series of experiments is, within the experimental error, equal to one half the maximum value computed in equation (9) on the assumption that negative electrons alone are effective. The same is true of the result of the earlier observations given in §24.

§37. For the same intrinsic magnetic intensity of rotation and the same material, the intensity of magnetization depends upon the shape of the rotating body and is less for a sphere than for a long cylinder.

[1] In the experiments of Lebedew (Ann. der Phys., 39, 1912, p. 840) the magnetic effect of rotating at high speed anyone of the several *non-magnetic* substances tried was nil, in conformity with the theory set forth in this paper. Lebedew's experiments were performed for a different purpose from that of this investigation, their object being to study the effect of centrifugal electron displacement, and the data he gives are insufficient for an exact calculation of the superior limit they give for the effect considered here. An approximate calculation, however, can be made and shows that the intensity of magnetization produced in Lebedew's non-magnetic bodies was not greater than about one hundredth of the value here obtained for steel, reduction being made to the same speed.

The earth's intensity of magnetization is about 0.08 c.g.s. unit, and its angular velocity is 1/86,400 r.p.s. If therefore our iron were spherical in shape and rotating at the speed of the earth, its intensity of magnetization would be less than the fraction $\frac{1.5 \times 10^{-6}}{86{,}400 \times 0.08}$, or 2×10^{-10}, the earth's intensity of magnetization.

Inasmuch, however, as we have no knowledge of the magnetic behavior of any substance under the conditions prevailing at more than a very small distance within the surface of the earth, it is still possible that this effect, which has the same sign as the earth's magnetization, may also account for its magnitude. It seems probable, however, that at least two other effects are prominently involved, viz., the centrifugal displacement and the thermionic displacement of electrons, which give, when rotation occurs, magnetic intensities in the direction required.

§38. SUMMARY.

1. On the electron theory it is shown that by a sort of molecular gyroscopic process a body of any substance set into rotation becomes the seat of a uniform intrinsic magnetic intensity parallel to the axis of rotation, proportional to the angular velocity, and (like the magnetization of the earth and the sun) directed, provided the effect of the negative electrons is preponderant, oppositely to the intensity which would be produced by an electric current flowing around the body in the direction of rotation. If the substance is magnetic, magnetization results; otherwise not. If only the negative electrons are involved, the intensity per unit speed has its maximum numerical value and is calculated to be

$$\frac{H}{n} = -7.1 \times 10^{-7} \frac{\text{gauss}}{\text{r.p.s.}}.$$

2. The theory of the converse effect (rotation by magnetization) is developed in a very simple manner.

3. An extended series of experiments on the magnetization of steel rods by rotation is described. All suspected sources of systematic error have been eliminated by the elaborate precautions taken, and it is believed that the effect predicted by theory has been discovered. The method of electromagnetic induction was used, a fluxmeter, whose deflections were read to tenths of mm. at the scale distance 8 m. (usually), being the chief measuring instrument. The intrinsic magnetic intensity of rotation, and the change of magnetic flux density, per unit speed were found to be about

$$-3.1 \times 10^{-7} \frac{\text{gauss}}{\text{r.p.s.}}$$

and

$$-1.9 \times 10^{-5} \left(\frac{\text{maxwells}}{\text{cm.}^2} \right) \text{per r.p.s.},$$

respectively. Experiments made by Lebedew for a different purpose are quoted as showing that non-magnetic substances are not magnetized by the process by which iron is here shown to be magnetized.

4. Experiments, incidental to the work, on the use of a fluxmeter when unidirectional and alternating flux changes occur either alone or superposed, and in the change of residual magnetization by torsion, are described.

5. Together with the change of flux proportional to the speed, another change proportional to the square of the speed has been found and accounted for by the radial expansion of the rod produced by rotation. In support of this view experiments are described in which the change of (residual) longitudinal flux through the rods was determined when the rods were longitudinally compressed and when the stress was removed.

6. The intensity of magnetization produced in the rotating iron per unit speed was about 1.5×10^{-6} c.g.s. unit per r.p.s. If the rod had been rotated at the speed of the earth, viz., 1/86,400 r.p.s., its intensity of magnetization would have been about 2×10^{-10} that of the earth. This, however, does not prove that the earth's magnetization may not be due largely to the effect in question, as we are entirely ignorant of the magnetic properties of all substances under the conditions prevailing within almost the whole of the earth. Other causes are mentioned as probably accounting for at least a part of the earth's magnetism. Schuster has suggested that an effect of the sort chiefly investigated in this paper may explain the secular variation as well as the mean magnetization of the earth.

I am indebted to Mrs. Barnett for a great deal of help throughout the experimental part of this work, and to Mr. Arthur Freund, mechanician in this laboratory, for constructing most of the special apparatus and for other mechanical assistance.

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ELECTROSTATIC MEASUREMENT OF ELECTRODE POTENTIALS.

BY ARTHUR W. EWELL.

THE determination of the difference of electrical potential of two electrodes dipping in a common electrolyte is simple and can be accomplished with a high degree of accuracy by several well-known methods. Measurement of the absolute potential difference between a single electrode and the electrolyte is difficult and the various methods which have been employed have yielded widely differing results.[1] Excellent bibliographies of this entire subject are accessible[2] and will not be repeated here. The papers of Borelius and Guyot referred to later are the only important memoirs which have appeared since these compilations.

HISTORICAL.

The different experimental methods for determining electrode potentials may be classified respectively as electro-capillary, endosmotic, capacity and electrostatic.

Mercury is naturally the electrode which has been employed for electrocapillary methods. Assuming that the absolute potential difference between mercury and an electrolyte results in the formation of a Helmholtz "double layer" which reduces the surface tension, Ostwald and others have concluded that when mercury in a capillary tube shows maximum surface tension the double layer is absent and that therefore the applied potential required to give the maximum surface tension is equal and opposite to the natural potential difference between mercury and the electrolyte. By somewhat similar reasoning mercury breaking into fine drops at the surface of the electrolyte was assumed to come to the same potential as the electrolyte and therefore the difference of potential between the reservoir supplying the drops and the mercury collecting at the bottom of the electrolyte was the absolute potential sought. The results by the two methods agree closely and give for the

[1] "Wir gegenwartig über die Komponenten der electromotorischen Kraft eines Elementes einfach gar Nichts wissen" (Chwolson).

[2] Palmaer, Zeit. f. phys. Chem., 59, 129, 1907. Potentialkommission d. deutsch. Bunsen Gess., Zeit. f. Electrochemie, 50, 818, 1908. Chwolson, Lehrbuch d. Physik, IV., I., 228, 1908.

absolute potential, if the solution contains normal concentration of mercury ions, about 1.05 volts, or when combined[1] with a "normal calomel electrode" give for the absolute potential of the latter 0.593 volts, the mercury of the electrode being positive with reference to the solution.[2]

Endosmotic methods have been chiefly developed by Billitzer.[3] He observed the motion of fine wires suspended in an electrolyte between parallel plate electrodes and also the motion of colloidal particles which dropped between the plates. The concentration of the electrolyte was varied until a difference of potential applied to the plates produced no motion of the wires or the particles and he concluded that they were then at the same potential as the solution. These experiments, with others of a similar nature, gave for the absolute potential of the tenth normal calomel electrode $-.125$ volt, the negative sign signifying that the mercury was at a lower potential than the calomel solution. Billitzer also obtained approximately the same results by methods depending upon the change of concentration in the vicinity of a dropping electrode, motion of a mercury meniscus, and motion of the electrolyte.

The results of Billitzer differed so greatly from the values given by capillary and dropping electrodes that portions of his work were repeated by Whitney and Blake,[4] and Goodwin and Sosman.[5]

These investigators either failed to obtain his results or interpreted them differently. Later Freundlich and Mäkelt[6] succeeded in duplicating certain of Billitzer's determinations, but failed to obtain his values when the conditions were somewhat varied. Billitzer answered these criticisms with explanations of their failures and arguments for the validity of his original results. He ascribed the difference between his values and those obtained by capillary electrodes to direct or indirect dependence of the surface tension upon the applied potential resulting in the maximum surface tension appearing not when electrode and electrolyte are at the same potential but when the electrode is at about 0.7 lower potential.[7]

The third method for evaluating absolute electrode potentials is also due to Billitzer,[8] and is based upon the assumption that changes in the

[1] If one single electrode potential is determined absolutely with reference to a particular electrolyte any other electrode may also be inserted in the electrolyte and its potential determined from the absolute potential of the first and the difference of potential of the two.

[2] Palmaer, Zeit. f. phys. Chem., 59, 129, 1907.

[3] Drude's Ann., II., 902, 1903. Zeit. f. phys. Chemie, 48, 513, 542, 1904; 48, 624, 1908.

[4] Jour. Am. Chem. Soc., 26, 1339, 1904.

[5] PHYS. REV., 21, 129, 1905.

[6] Zeit. f. Electrochemie, 15, 161, 1909.

[7] Zeit. f. phys. Chemie, 51, 167, 1905. Zeit. f. Electrochemie, 12, 281, 1906; 14, 624, 1908; 15, 439, 1909.

[8] Zeit. f. phys. Chemie, 51, 167, 1905.

potential difference between the two elements of the electrode-electrolyte interface will produce changes in the distance between the two oppositely charged layers and consequently changes in the capacity. Billitzer found evidence in the works of Scott[1] and Kruger[2] of marked abnormality in the capacity at the potential which he had determined as the absolute potential by endosmotic methods. The writer, however, after careful study of the original observations is not convinced that these abnormalities are demonstrated beyond the limits of experimental errors.

The fourth method of determining absolute electrode potentials consists in direct electrostatic measurement. A large amount of work was done by this method previous to 1890, references to which may be found in the bibliography in Chwolson.

The observations of Exner and Tuma[3] were the only ones which were reproduceable and fairly consistent, *i. e.*, which gave for two different electrodes values of the absolute potentials whose algebraic difference was their known potential difference. Their method was similar to Kelvin's water dropper with the water replaced by mercury and the cylinder lined inside by filter paper saturated with the electrolyte which was connected by a wick with the electrolyte in which the electrode dipped. Their results were in rough agreement with those of Billitzer. Certain assumptions in their method may however involve serious systematic errors, which are about to be investigated in this laboratory. The only recent electrostatic work known to the writer is a paper by Borelius describing measurements of the relative, not absolute, potentials in the vicinity of an electrode and its electrolyte.[4]

This uncertainty in the absolute values of electrode potentials led the writer to begin several years ago an attempt to obtain accurate and reliable measurements by electrostatic methods. The final apparatus alone will be described and only the observations made during the past eight months will be presented.

APPARATUS.

Fig. 1 is a schematic sketch of the final apparatus. The electrolyte was usually contained in a flask A, 50 to 150 cm.^{3} capacity mounted on a platform supported by legs a of fused quartz. The outside of the flask except the neck was coated with a thin metallic film to which connection was made by the wire b. The latter could either be connected by the movable wire c to the Dolezalek electrometer C or, by the movable wire d with the wire e which could be connected to the earth or to a

[1] Wied. Ann., 67, 388, 1899.

[2] Zeit. f. phys. Chemie, 45, I., 1903.

[3] Wien. Akad. Sitz. Berich., 97, 917, 1889.

[4] Ann. d. Physik, 42, 1129, 1913.

source of known potential. The portions f of the threads by which c and d were raised and lowered were of fused quartz. The entire apparatus including the water battery g for charging the electrometer needle was surrounded by grounded wire netting. The electrode B (about 1 mm. in diameter) could be moved up and down and was usually grounded.

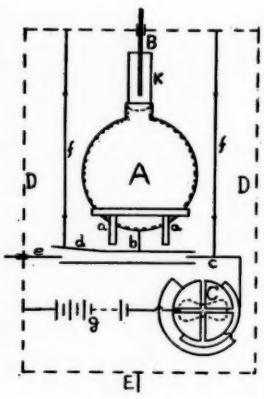

Fig. 1.

The theory of the method was as follows: If the electrolyte was initially at zero potential, upon introduction of the earthed electrode B it would acquire a potential equal to its natural absolute potential with reference to the particular electrode. A charge at somewhat different potential would be induced upon the metal film upon the outside of A, the deflection of the electrometer would be proportional to this potential, and, by a calibration described later, the potential of the electrolyte could be ascertained.

The procedure of an observation was as follows: The electrometer and the outside of the flask were earthed by the wires b, c, d and e, the latter being connected to earth. The electrolyte was poured into A. The neck k and the legs a were then cleansed with dilute alcohol and heated with a Bunsen burner to remove any moisture. The earthed electrode B was then lowered until it dipped in the electrolyte and immediately withdrawn. No readings were taken because the introduction of the electrolyte and the heating of the neck often produced static charges of many volts potential. After standing undisturbed several hours d was disconnected from e, and the electrometer reading was observed on a scale 160 cm. from the mirror. The earthed electrode B was then lowered until it dipped in the electrolyte and the accompanying deflection of the electrometer was observed from which, as explained later, the absolute potential of the electrolyte was calculated.

The following readings are those of a typical experiment.

Oct. 5, 1914. Flask e^1. Normal $ZnSO_4$ solution.

Electrometer Readings.

After heating neck and legs	49.5
Outside film and needle earthed	48.1
Outside film and needle insulated.	
Earthed zinc electrode dipped in electrolyte	39.9
One minute later	40.0
Fourteen minutes later	40.2
Thirty-two minutes later	41.0

[1] See below.

SOURCES OF ERROR.

1. *Poor Insulation.*—The loss of charge rarely exceeded 2 per cent. per minute. Since the complete period of the electrometer needle was about four seconds and its motion was considerably damped, a reading required only a small fraction of a minute. Occasionally the leakage was excessive and the observation was rejected. If the metal film on the outside was different from the electrode and the electrolyte had stood for some days in a glass flask there was occasionally evidence of conduction through the glass. When such conductivity was present the electrometer would not at once come to a definite reading when the earthed electrode was introduced but slowly drifted, and the final resting point of the needle when the electrode was left in the electrolyte differed from the zero position. Also there was a slow drift to this latter position if the needle was momentarily earthed and then insulated, the electrode remaining in the electrolyte. Quartz flasks never showed such conductivity. In particularly cold, dry weather surface leakage was inappreciable, but such static charges appeared when any part of the apparatus was disturbed that observations were impossible.

2. *Volta Effects.*—We are seeking the absolute difference of potential between an electrode and the surrounding electrolyte. The writer believes that the apparatus and procedure outlined above eliminates so-called Volta potentials or effects due to possible differences in potential in the air immediately surrounding electrode and electrolyte. As proof of the correctness of his belief, the electrolyte was successively replaced by mercury in each of the flasks employed and the deflections of the electrometer were observed when earthed wires of different metals (Cu, Ag, Zn, Ni, Fe) were dipped into the mercury. In every case the potential observed was less than 0.1 volt and usually it was inappreciable.

3. *Initial Static Charges on Electrolyte.*—As mentioned above, such charges were usually observed when the flask was first filled and the neck of the flask was heated. It was assumed that after several hours these static charges largely disappeared and that the potential of the electrolyte differed little from zero (the potential of the cage). The deviations in the observations are probably largely attributable to this cause. It was not possible to further reduce errors from this source by delaying observations because errors of the first type usually began to appear. The fact that mercury in the different flasks and with different electrodes showed practically no potential is further evidence against this source of error being serious.

4. *Influence of Material of Flask and Metal Film.*—To eliminate any such specific influence different materials were used which are designated as follows in subsequent tables.

a = quartz flask No. 1 with outside film of silver.[1]
b = quartz flask No. 1 with outside film of aluminum.[2]
c = quartz flask No. 2 with outside film of silver.
d = quartz flask No. 2 with outside film of aluminum.
e = glass flask No. 1 with outside film of silver.
f = glass flask No. 1 with outside film of aluminum.
g = glass flask No. 2 with outside film of silver.
h = glass flask No. 2 with outside film of aluminum.

Observations with paraffin coated vessels and vessels of other shapes are described later.

EXPERIMENTAL RESULTS.

The following table gives all the observations made during the past months with narrow neck flasks with the final form of apparatus except a few observations which were discarded because of evidence of errors of the first type mentioned above. At the foot of each column is stated the mean with its probable error, except for Cu – $CuSO_4$ where the variation in the readings is so great that little quantitative importance can be given the results.

CALIBRATION.

The algebraic difference in the mean deflections for zinc and silver is 11.67. The difference of potential of zinc and silver electrodes, each in a normal solution is 1.541 volts.[3] Dividing one by the other gives 7.50 divisions per volt as the calibration constant of the system. This constant was checked by substituting mercury for the electrolyte and observing the electrometer deflection due to the charge induced on the outside of the flask when the potential of the mercury was changed by known steps by means of the electrode B.

This method of calibration gave values for this constant, which for narrow neck vessels were often as much as 15 per cent. greater than the values calculated as just described while for wide necked vessels (see below) both methods of calibration gave almost identical results. This difference was to be expected since the outside metallic coating was nearer the wire when the neck was narrow and this wire had a like charge in the direct calibration with mercury and a charge of opposite sign in the method of calibration by known differences of potential, and in all the actual experiments. The mercury calibration proved that the deflection of the electrometer was proportional to the potential within 0.2 per cent. up to 1.43 volts and that there was no appreciable asymmetry in the deflections on the two sides.

[1] Chemically deposited upon flask.

[2] Aluminum foil attached by a very thin film of shellac.

[3] Wilsmore's Tables, Winkelmann's Handbuch, IV., I., p. 856.

TABLE I.

Zn $n ZnSO_4$	Ag $n AgNO_3$	Cu $n CuSO_4$	Zn $n/2 H_2SO_4$	Cu $n/2 H_2SO_4$
a +7.8	*a* −2.0	*a* 0.0	*a* +9.2	
" 6.3	" .5	" +2.3	" 7.0	*a* +2.2
" 6.0	" 3.5	" + .8	" 8.7	" 0.0
" 8.8	" 3.5	*e* +3.0	" 8.3	" +3.5
" 9.1	" 4.3	*g* + .3	*e* 9.7	" +2.1
" 7.3	" 3.6	" +1.1	" 9.3	" +1.1
b 8.0	" 3.0	*f* −2.9	" 9.2	" − .8
" 7.9	" 2.8	" −2.3	" 8.8	" − .3
" 8.4	" 2.8	" − .5	" 7.9	" + .2
c 7.0	" 2.4	*c* − .3	*f* 7.6	*e* +1.0
" 7.4	*b* 2.0		" 7.7	" +1.1
" 6.5	" .7		" 7.6	" +1.2
g 8.2	" 3.2		*g* 8.8	" + .6
f 10.6	" 3.5		" 8.7	" + .7
" 6.9	" 3.0		" 8.7	*f* − .8
g 8.5	" 4.7		" 11.6	" − .8
" 9.6	" 4.4		" 10.9	" − .7
" 9.0	" 3.3		" 8.1	*g* + .5
" 8.0	*e* 5.4		" 7.1	" + .4
" 9.2	" 4.9		" 10.2	" + .4
f 7.7	" 4.8		" 9.2	" +3.1
" 6.3	*f* 3.1		" 9.3	" +2.9
c 7.2	" 8.3		" 9.6	" +1.3
	h 2.4		*f* 6.5	" +1.0
	g 1.8		" 7.3	" +1.4
	f 6.8		" 8.7	" +2.5
	" 7.3		" 5.5	" +1.8
	" 4.6		" 5.4	" +2.3
	c 4.1		" 5.7	*f* −1.5
			" 5.7	" − .7
			" 7.3	" −2.1
			" 7.2	" −2.5
				" −2.3
				" −1.9
				" −2.0
				" − .3
				" − .2
+7.90±.16	−3.67±.19	+0.18	+7.84±.20	+0.37±.19

FINAL RESULTS.

Dividing the mean deflection by this constant 7.50 we find for the potential of zinc in normal zinc sulphate solution − 1.05 ± .02 volts, and for zinc in half normal sulphuric acid solution − 1.04 volts. For silver in normal silver nitrate solution we find + .49 ± .03 volt and for

copper in normal copper sulphate solution — .02 volt and in half normal sulphuric acid solution — .05 volt.[1]

Observe that these results are mutually consistent. The respective potentials of zinc and copper are very nearly the same in both electrolytes, as would be expected. The differences of potential of copper and zinc are 1.07 and 1.09 volts respectively which compare well with the difference of 1.099 in Wilsmore's table.

WIDE NECK VESSELS.

To make certain that the results were not influenced by the degree to which the outside metal film approached a "closed" conductor, observations were made both with test-tubes 2.5 cm. in diameter 20.5 cm.

TABLE II.

Test-tube.	Beaker.	Test-tube Covered Inside with Paraffin.
Ag. — 4n $AgNO_3$		Ag — n $AgNO_3$
3.1	3.8	4.0
3.6	3.6	5.0
3.2	4.0	4.3
3.6	3.2	3.4
3.4	3.3	3.9
3.3	3.0	4.1
3.3	——	3.4
3.6	Mean, 3.48	2.7
4.2	(0.40 volts)	——
3.2		3.85
——		(0.50 volts)
Mean, 3.47		
(.38 volts)		
Zn — 2.5n $ZnSO_4$		Zn — n $ZnSO_4$
9.9	11.6	8.1
10.8	9.9	7.9
11.1	8.8	8.2
11.1	——	——
10.6	Mean, 10.1	Mean, 8.1
10.1	(1.15 volts)	(1.05 volts)
10.1		
10.4		
11.0		
——		
Mean, 10.5		
(1.16 volts)		

[1] The signs in Table I. apply to the electrolyte since its potential rather than that of the electrode was under observation. The electrode potential will obviously have the opposite sign.

long, covered with an outside silver film for 7.5 cm. from the closed end, and with a beaker about 7 cm. high, 4 cm. in diameter at the top and 3 cm. in diameter at the bottom with an outside silver coating extending to within about 3.5 cm. of the top. One test-tube was covered upon the inside with paraffin to secure further evidence that the results were independent of the insulating medium. The electrometer readings are given in Table II.

The potentials stated in parentheses were calculated as explained above. Allowing for the difference in concentration, they agree closely with the previous results and show that the values obtained are independent of the shape of the vessel. The results in the last column show that a coating of paraffin inside the vessel does not affect the potential. Similar observations were made with a number of other test-tubes with like results. Further experimental confirmation of certain of the above figures will be found when considering the effect of concentration. Table III. gives a comparison of the writer's results

TABLE III.

	n/10 Calomel Electrode.	Silver.	Zinc.
Wilsmore and Palmaer	+.573	+1.008	− .533
Ewell	(+.06)	+ .49	−1.05

with the values commonly found in tables (Wilsmore's) except that the latter are decreased by .04 volt to correspond to Palmaer's more recent value of the potential of the tenth normal calomel electrode (.573 volt in place of Wilsmore's value of .613).

INFLUENCE OF CONCENTRATION OF ELECTROLYTE.

The final form of apparatus is illustrated in Fig. 2. The glass flasks *A* and *A'* contained two concentrations of the same electrolyte. The silver film on the outside of one flask was connected to one pair of quadrants of the electrometer and the other film to the other pair of quadrants. The wires *b*, *b'* permitted grounding of the films and quadrants. An observation was preceded by preliminary steps similar to those previously described. The deflection of the electrometer was observed when one earthed electrode *B* was dipped into the electrolyte and

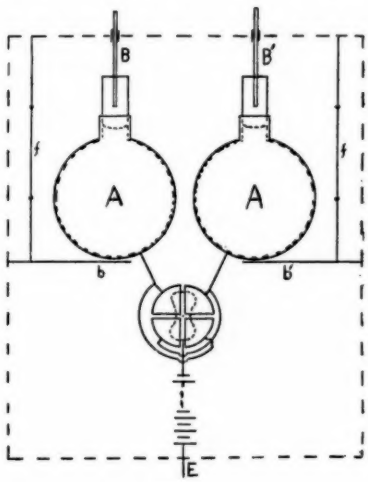

Fig. 2.

then when the other electrode B' was also dipped into the other electrolyte. From the deflection with both earthed electrodes in their respective solutions the difference of potential of the two electrolytes was determined. Lack of symmetry was eliminated by changing the order of introduction of the electrodes and by exchanging the two concentrations of the solution between the two flasks. Table IV. gives the final readings with silver nitrate and zinc sulphate.

TABLE IV.

Electrode.	Electrolyte.	Deflection of Electrometer upon Dipping Earthed Electrode into Flask Connected with		Both Electrodes in their Respective Solutions.
		First Pair of Quadrants.[1]	Second Pair of Quadrants.	
Ag.....	$4n$ $AgNO_3$ in first flask.....	−3.1	+4.4	+1.3
		−2.2	+2.7	+ .5
	$n/250$ $AgNO_3$ in second flask.....	−2.5	+4.3	+1.8
		−3.2	+3.6	+ .4
		−2.6	+3.2	+ .6
		−2.6	+2.8	+ .2
		−1.8	+2.9	+1.1
	$n/250$ $AgNO_3$ in first flask.....	−2.9	+ .8	−2.1
		−2.7	+1.7	−1.0
	$4n$ $AgNO_3$ in second flask.....	−2.1	+1.4	− .7
		−3.0	+2.5	− .5
		−1.7	+ .9	− .8
		−3.6	+2.9	− .7
Zn.....	$2n$ $ZnSO_4$ in first flask.....	+4.7	−3.4	+1.3
		+5.2	−4.0	+1.2
	$n/500$ $ZnSO_4$ in second flask.....	+5.1	−4.0	+ .9
		+5.7	−4.5	+1.2
		+5.0	−3.9	+1.1
		+5.4	−4.6	+ .8
	$n/500$ $ZnSO_4$ in first flask.....	+3.7	−5.0	−1.3
		+4.4	−5.0	− .6
	$2n$ $ZnSO_4$ in second flask.....	+4.0	−4.8	− .8
		+3.8	−4.9	−1.1
		+2.9	−4.4	−1.5

[1] The first pair of quadrants is the pair to which the flask was connected in the work described earlier, and the first flask is the flask connected to these quadrants.

The lower concentration of silver nitrate gives the greater deflection the mean excess being 0.90 which divided by 7.50 gives 0.12 volt. The sign of the deflection shows that the weaker solution is at a lower potential, and therefore a silver electrode in $4n$ $AgNO_3$ solution in a glass

vessel is about 0.1 volt lower potential than a silver electrode in $n/250$ $AgNO_3$ solution.

A concentration cell

$$Ag - 4n\ AgNO_3 - n/250\ AgNO_3 - Ag$$

gave by potentiometer measurement an E.M.F. of 0.13 volt, the silver in the *stronger* solution being at the higher potential, while by Nernst's theory this electrode should have an excess potential of 0.16 volt.

The electrode in the stronger $ZnSO_4$ solution shows a mean excess deflection of 1.07 divisions. A zinc electrode in $2n$ $ZnSO_4$ solution in a glass vessel is therefore about 0.14 volt more negative than a similar zinc electrode in $n/500$ $ZnSO_4$ solution. Borelius observations referred to earlier, gave approximately 0.17 difference in the potentials induced in a conductor upon approaching successively solutions of zinc nitrate containing earthed zinc electrodes when the concentration ratio was 1,000, and the sign of the induced potential indicated that the stronger solution (or the region about it) was more negative. In the opinion of the writer it is not however possible to differentiate in Borelius observations between Volta potentials and the true electrolytic potentials which we are here investigating.

For these observations upon the influence of concentration, it was necessary to keep the electrolyte in the flasks for considerable lengths of time and the possibility of some electrolytic conduction through the glass reduces somewhat the dependence which may be given the results. However, although there is a wide variation in the above small differences, taken as a whole, they indicate that when a single electrode dips in a homogeneous electrolyte containing ions of the metal in a glass vessel, the potential of the metal decreases as the concentration of the solution increases. This is opposite to the theory of Nernst, which has often been approximately substantiated for the quite different cases of *two* electrodes, either both dipping in a non-homogeneous solution, or one dipping in the solution and the other separated from the solution by ionized air.[1]

If from an external source the film on the outside of one flask was raised to a known potential, the other flask and its quadrants being insulated, the deflection was 0.59 of the deflection when this pair of quadrants was earthed. The mean deflection with $4n$ AgNO was

$$2.17 = \frac{2.17}{.59 \times 7.50} = -.49 \text{ volt.}$$

The mean deflection with $2n$ ZnSO = 5.02 = 1.11 volts. Both of these potentials are seen to be in fair agreement with the results of direct measurement.

[1] Guyot, C. R., 153, 867, 1911.

CONCLUSIONS.

A new method for measuring absolutely single electrode potentials has been developed. The results obtained indicate that the potential of a metal in a homogeneous normal solution of one of its salts is about 0.5 volt more negative than the potential deduced from electrocapillary phenomena and usually found in tables. Increasing the concentration of a salt solution contained in a glass vessel appears to lower the potential of an electrode of the metal of the salt.[1]

The writer wishes to express his indebtedness to Mr. C. G. Cummings for assistance in the preliminary observations.

WORCESTER POLYTECHNIC INSTITUTE,
April 24, 1915.

[1] After this paper had been sent to the editors, a second paper of Borelius (Ann., 22, p. 927, 1914) came to the writer's attention, in which the experiments described in the earlier paper were repeated with electrolytes contained in paraffin-coated flasks connected with a calomel electrode by a syphon. In contrast to the previous results, these later observations gave values for the potentials outside the glass vessels which were in accord with Nernst's theory. The writer, with the assistance of Mr. A. G. Aldrin, has repeated the concentration observations described above using vessels of various shapes of clean glass, glass vessels covered inside with paraffin, and glass vessels covered with shellac. The results with clean glass have been similar to those described above, *i. e.*, opposite to Nernst's theory. A limited number of observations with coated glass have usually given qualitative agreement with the theory. Further investigation is in progress.

LEAKAGE OF GASES THROUGH QUARTZ TUBES.

BY E. C. MAYER.

HISTORICAL.

BESIDES the work which has been undertaken on the investigation of the flow of gases through quartz and glass, a considerable amount of attention has been given to the determination of the absorption and diffusion of gases through metals, principally iron, palladium and platinum. Among the most important work on metals may be mentioned that of Lockyer,[1] Travers,[2] Ramsay and Travers,[3] Winkelmann,[4] Graham,[5] Beetz,[6] Hoitsema,[7] Schmidt,[8] Richardson, Nicol, and Parnell,[9] Richardson,[10] and Dorn.[11] The gases investigated included hydrogen, helium, and argon.

That quartz is permeable to gases has been found by Villard,[12] Jaquerod and Perrot,[13] and Richardson and Ditto.[14]

OBJECT.

The purpose of the present investigation was to determine the leakage of hydrogen, nitrogen and oxygen through a quartz tube, and to see whether there existed any relation between the amount of leakage and the pressure and temperature of the gas.

APPARATUS.

The apparatus employed is shown in Fig. 1. The manometer consists of the long glass tube *ABCDE* joined to the cylindrical reservoir *GH* by

[1] J. N. Lockyer, Proc. Roy. Soc., 1895, vol. 58, p. 193.

[2] M. W. Travers, Proc. Roy. Soc., 1896, vol. 60, p. 449.

[3] Ramsay and Travers, Proc. Roy. Soc., 1897, vol. 61, p. 267.

[4] A. Winkelmann, Ann. d. Physik, 1901, vol. 311, p. 104; 1902, vol. 313, p. 388; 1905, vol. 321, p. 773; 1906, vol. 324, p. 1045.

[5] T. Graham, Pogg. Ann., 1869, vol. 136, p. 317; 1866, vol. 129, p. 580; 1866, vol. 129, p. 602.

[6] W. Beetz, Sitzungsber. der Bayer. Akad., 1878, p. 157.

[7] C. Hoitsema, Zeitschr. f. physik. Chem., 1895, vol. 17, p. 40.

[8] G. N. St. Schmidt, Ann. d. Physik, 1904, vol. 318, p. 747.

[9] Richardson, Nicol, and Parnell, Phil. Mag., 1904, vol. 8, pp. 1–29.

[10] O. W. Richardson, Cambridge Phil. Soc. Proc., 1905, vol. 13, p. 27.

[11] E. Dorn, Phys. Zeitschr., 1906, vol. 7, p. 312.

[12] P. Villard, Comptes Rendus, 1900, vol. 130, p. 1752.

[13] Jaquerod and Perrot, Archives des Sciences, 1904, vol. 18, p. 613; 1905, vol. 20, p. 454.

[14] Richardson and Ditto, Phil. Mag., 1911, vol. 22, p. 704.

means of black pressure tubing *FG*. *F* is a one-way stopcock. *DE* is inclined to the horizontal in the manner shown so that small differences of pressure are easily detected. A difference of pressure of 1 cm. of the gas under consideration is accompanied by a movement of the mercury in *DE* of 2.99 cm.

The eudiometer consists of a glass tube *JK* calibrated in grams of water correct at 15° C. This is joined to a tube *LM* at *L*. *R* and *N* are one-way stopcocks. *Q* is a mercury reservoir, joined to the eudiometer by means of the black pressure tubing *NP*.

Heating was obtained by means of an electric furnace. Temperature

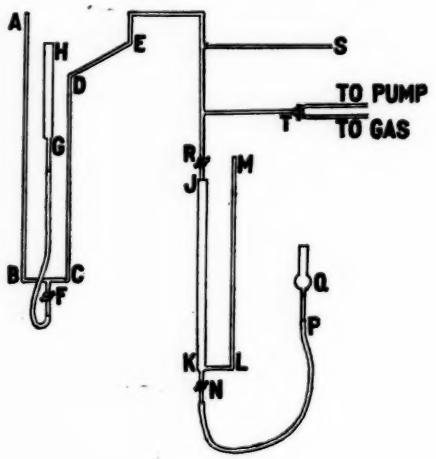

Fig. 1.

was measured by a Paul unipivot pyrometer. Specimens of Jena glass and quartz were joined to the main apparatus at *S*. The specimen of transparent quartz was 16.5 cm. long, 1.9 cm. internal diameter, 0.05 cm. wall thickness. The Jena glass specimens were of approximately the same dimensions except that they had a wall thickness of 0.18 cm.

PROCEDURE.

The following procedure was adopted when it was desired to take a determination with the gas under atmospheric pressure. Connecting to the pump by means of stopcock *T*, the entire apparatus was exhausted; then turning *T* to the next position, gas was introduced. This was repeated twice so that the apparatus should be well washed with the gas. *R* was then closed and the gas trapped in the eudiometer could be placed under any pressure desirable by raising or lowering *Q*. This entire procedure was done only after the furnace had been maintained steadily at

the desired temperature for at least one hour. The gas passed through a calcium chloride drier before entering the apparatus. Cock T was now closed. GH was raised so that the mercury stood somewhere in the upper part of AB and in the lower part of DE (at somewhat lower level than mercury in AB). F was then closed. By referring to a calibration curve, an amount that GH should be raised was noted, so that after having raised GH this amount, and then opened F, mercury in DE would rise to same level as mercury in AB. If mercury would not rise all the way to this level, R was opened until sufficient gas had passed from main apparatus into eudiometer, and mercury in DE did come to this level. R was then closed. Mercury in AB, GH and DE would then be at the same level, and gas in apparatus would be under atmospheric pressure. Cock F was now closed, to remain closed throughout the determination.

The procedure was entirely analogous if the gas was to be under an initial pressure greater or less than one atmosphere.

If gas escaped by absorption or diffusion through the specimen, mercury in DE rose. Such movements were read to an accuracy of 0.1 cm. and estimated to 0.01 cm. Readings were reduced to 76 cm. pressure and 15° C.

THEORY.

Before discussing the data, it seems desirable to determine what results one may be led to expect from a theoretical consideration of the flow of gas in one direction, assuming an infinite solid (see Fig. 2) with two parallel plane faces whose distance apart is c. It may be shown that a small increment of gas dm flowing through a slab of thickness dx in a time dt is given by

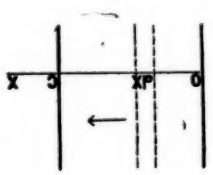

Fig. 2.

$$dm = -a^2 q \frac{\partial u}{\partial x} dt, \tag{1}$$

where q is the cross-section of the slab, $\partial u/\partial x$ the pressure gradient, a^2 the constant of flow. a^2 is the quantity of gas traversing unit area of the slab in unit time when the pressure gradient is unity.

Let the origin be in one face and the axis of X perpendicular thereto. Consider the initial pressure in the solid any function of x and suppose the pressure of the left-hand face of the solid is a constant β, and the right-hand face at zero pressure; we must solve the equation

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}. \tag{2}$$

We find[1]

$$\mu = \frac{2}{c}\sum_{m=1}^{m=\infty}\left[\epsilon^{-\frac{a^2m^2\pi^2}{c^2}t}\sin\frac{m\pi}{c}x\int_0^c f(x)\sin\frac{m\pi}{c}xdx\right] + \beta\left[\frac{c-x}{c} - \frac{2}{\pi}\sum_{m=1}^{m=\infty}\frac{1}{m}\epsilon^{-\frac{a^2m^2\pi^2}{c^2}t}\sin\frac{m\pi x}{c}\right] \tag{3}$$

as our required solution.

Since $u = f(x)$ when $t = 0$, if $u = 0$ when $t = 0$, $f(x) = 0$ when $t = 0$.

Differentiating[2] u with respect to x and integrating[3] with respect to t between the limits $t = 0$ and $t = T$, we obtain

$$M = a^2q\frac{\beta}{c}T - \frac{2\beta qc}{\pi^2}\left[\left(\sum_{m=1}^{m=\infty}\frac{1}{m^2}\epsilon^{-\frac{a^2m^2\pi^2}{c^2}T}\right) - \frac{\pi^2}{6}\right], \tag{4}$$

where M is the total mass of gas which has passed in a time T through the left-hand face of the solid.

If $f(x)$ is not zero when $t = 0$, but if $f(x) = b$, constant, then it may be shown that M becomes

$$M = a^2q\frac{\beta}{c}T - \frac{2\beta qc}{\pi^2}\left[\left(\sum_{m=1}^{m=\infty}\frac{1}{m^2}\epsilon^{-a^2\frac{m^2\pi^2}{c^2}T}\right) - \frac{\pi^2}{6}\right] + \frac{4bqc}{\pi^2}\left[\left(\sum_{m=1}^{m=1,3,5\cdots}\frac{1}{m^2}\epsilon^{-a^2\frac{m^2\pi^2}{c^2}T}\right) - \frac{\pi^2}{8}\right]. \tag{5}$$

If $u = \beta$ when $x = 0$, $u = \gamma$ when $x = c$, and if $u = f(x) = b$, constant, when $t = 0$, M becomes

$$M = a^2q\frac{(\beta-\gamma)}{c}T - \frac{2(\beta-\gamma)}{\pi^2}qc\sum_{m=1}^{m=\infty}\left[\frac{(-1)^m}{m^2}\epsilon^{-a^2\frac{m^2\pi^2}{c^2}T} + \frac{\pi^2}{12}\right] - \frac{4(\beta-b)}{\pi^2}qc\sum_{m=1}^{m=1,3,5\cdots}\left[\frac{1}{m^2}\epsilon^{-a^2\frac{m^2\pi^2}{c^2}T} - \frac{\pi^2}{8}\right]. \tag{6}$$

RESULTS.

In the experiments with Jena glass, there were no indications of leakage. It was impossible to use higher temperatures than 800° C. as the Jena glass became too soft. Also at the higher temperatures it was not feasible to use pressures much lower or higher than one atmosphere because of the softness of the glass. From the results it seems probable that Jena glass is not permeable to the common gases at moderately high temperatures.

[1] W. E. Byerly, Fourier's Series and Spherical Harmonics, 1893, pp. 38, 105.

[2] Byerly, p. 68. E. Goursat and E. R. Hedrick, Mathematical Analysis, 1904, vol. 1, p. 365. H. S. Carslaw, Fourier's Series and Integrals, 1906, pp. 55, 62.

[3] Goursat and Hedrick, p. 364.

A series of determinations at atmospheric pressure and room temperature were taken to see whether hydrogen or oxygen leak through quartz with the result that no leakage was found.

The results for hydrogen at temperatures 330°, 430°, 550°, and 710° C. are shown by the curves, Figs. 3 to 6. Time in minutes is plotted as

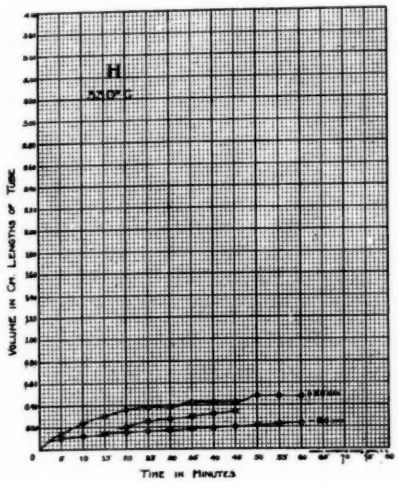

Fig. 3.

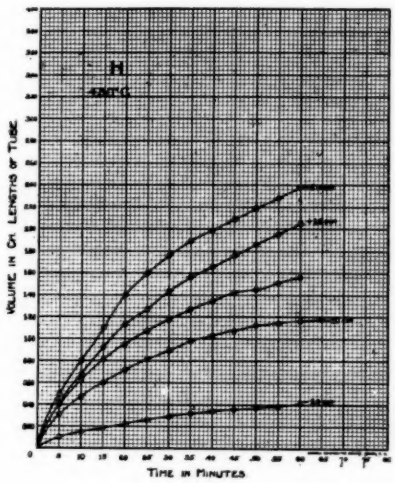

Fig. 4.

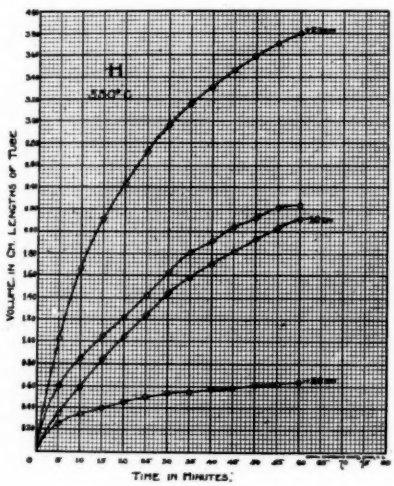

Fig. 5.

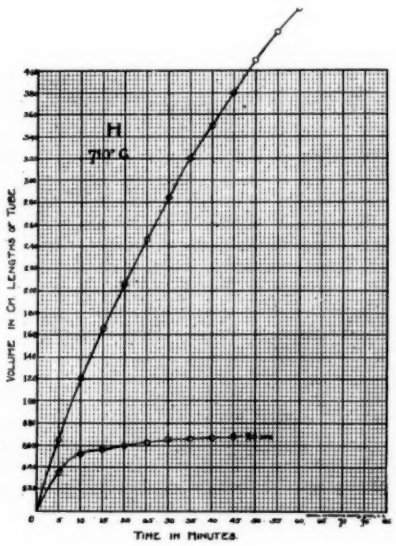

Fig. 6.

abscissas and volume of leakage reduced to 15° C. and 76 cm., in centimeter lengths of the tube *DE* as ordinates. The pressures used were

20 cm. above atmospheric pressure, 10 cm. above, atmospheric pressure, 10 cm. below, and 20 cm. below. Curves are designated to show under what pressure they were obtained. Determinations were taken for one hour.

One centimeter length of the tube *DE* corresponds to a volume of 0.217 cu. cm.

From an inspection of the curves, it is seen that temperature remaining constant, leakage increases with an increase in pressure. Furthermore it appears that the gas leaks more rapidly at the beginning of a determination than later. With the apparatus in the present work it is impossible to distinguish between the loss of gas by diffusion and by absorption by the walls of the specimen. However it is believed that the loss by absorption had been reduced to a minimum as the specimen was always maintained at the desired temperature for at least one hour before readings were taken, and furthermore the apparatus was washed with the gas twice immediately before the final gas was introduced for experimentation.

If curves are plotted to show the leakage of hydrogen under like conditions of pressure, it will be noted that the leakage increases with temperature, the increase being considerably more marked at the higher pressure.

The results for oxygen at temperatures 280°, 420°, 550° and 690° C. are shown by curves, Figs. 7 to 10.

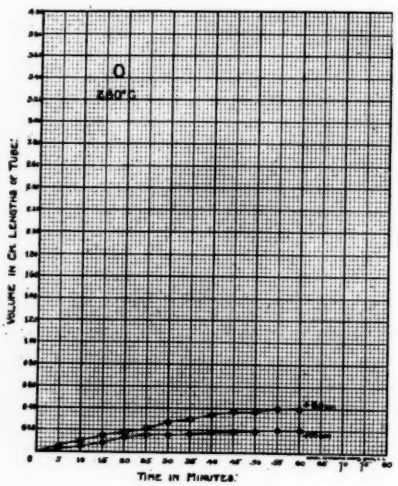

Fig. 7.

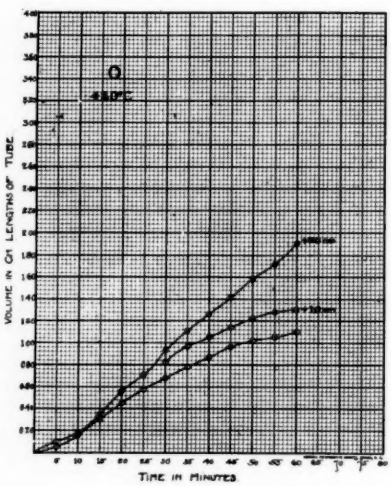

Fig. 8.

Among the curves no data are given corresponding to pressures smaller than one atmosphere. Under these conditions not a single trace of leakage of oxygen was detected.

The data for oxygen, like that for hydrogen, show an increase in leakage with increase in pressure at constant temperature, and a greater leakage with increase in temperature, pressure remaining constant.

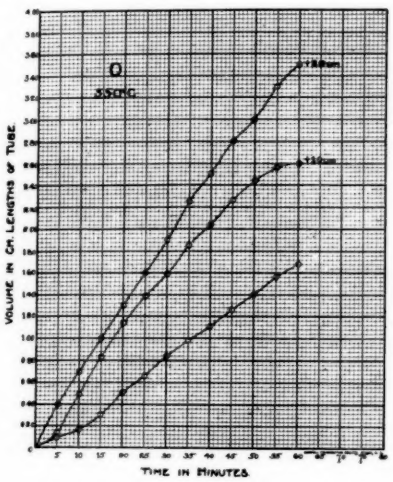

Fig. 9.

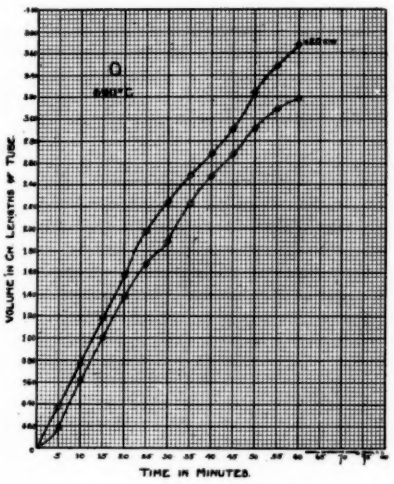

Fig. 10.

The data for nitrogen for temperature 430°, 550° and 700° C. are plotted in Figs. 11 to 13. No leakage of the gas was detected at the

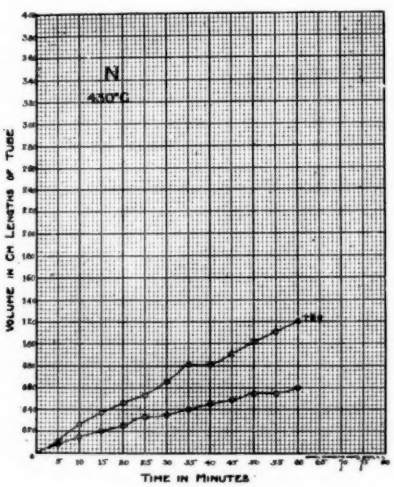

Fig. 11.

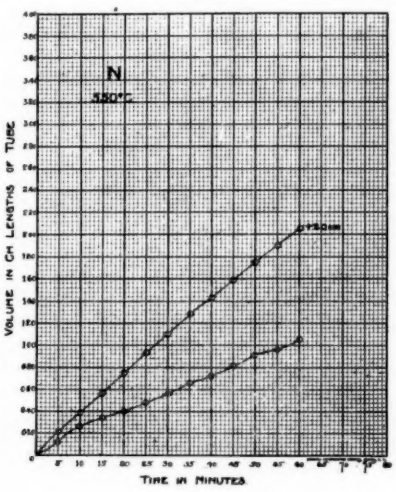

Fig. 12.

lowest temperature of about 300° C. as in the case of hydrogen and oxygen, nor was there any indication of leakage corresponding to a pressure lower than one atmosphere. Here again, at constant tempera-

ture, leakage increase with an increase of pressure, and at constant pressure, leakage increases with an increase in temperature.

Curves, of which Fig. 14 is an example, are interesting, as they show the relative leakage of hydrogen, oxygen and nitrogen under like conditions of pressure and approximately like conditions of temperature. In every instance, it is seen that hydrogen leaks most rapidly, and nitrogen least rapidly.

The theoretical treatment outlined above furnishes equations (4),

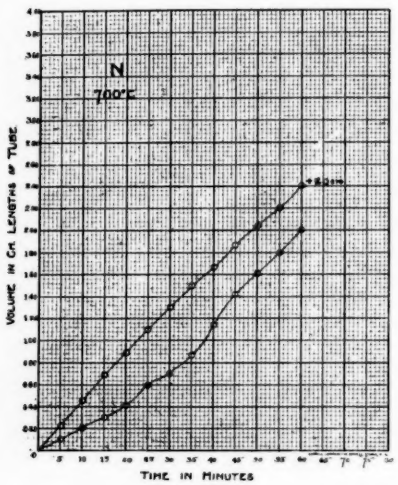

Fig. 13.

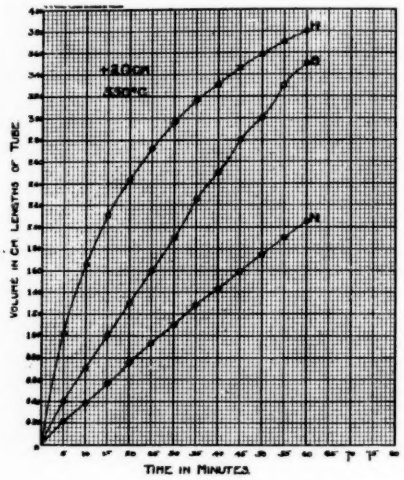

Fig. 14.

(5) and (6), by means of which it is possible to check the experimental data. In the case of hydrogen leaking into the atmosphere, if $b = 0$, equation (4) may be applied.

By equation (4) and the leakage for hydrogen at one atmosphere, 430° C., and after 60 minutes, an approximate value (0.000003474) of a^2 was computed and this used to calculate data for curves for hydrogen at 430° C. The first four terms only of the infinite series in equation (4) were taken. M was expressed in centimeter lengths of the tube DE (see Fig. 1), q in square centimeters, β in meters of mercury and T in minutes.

A comparison of the curves so determined with those obtained experimentally does not indicate a close agreement, particularly for a pressure 20 centimeters below atmospheric.

If $b = \gamma =$ constant, the curves computed on basis on equations (4) and (6) will be similar.

If b constant (to be determined from data as stated below), and $\gamma = 0$, equation (5) may be applied. Now if curves be drawn showing

the relation between the quantity of hydrogen leaking at 430° C. (for example, in equal time), and the pressure, the positive X-intercept indicates that under a pressure represented by this intercept, hydrogen does not leak, irrespective of the time. If b is taken equal to 25 centimeters (the average positive X-intercept) and an approximate value (0.00000917) of a^2 is computed, and curves plotted from calculations by means of equation (5), these curves again do not check with those found experimentally, the largest deviation occurring for a pressure of 56 centimeters.

However it appears hardly probable that hydrogen exerting a definite pressure b exists in the interstices of the quartz as would be assumed by the theory summed up in equation (5). If quartz does contain occluded hydrogen, it seems reasonable to assert that this gas is in equilibrium and forms, as it were, a constituent part of the quartz itself. So that of the two ideas proposed, the one based on equation (4) appears the more likely. Inasmuch as this does not explain the results adequately, we cannot offer any positive explanation of the leakage of hydrogen through quartz. Possibly some other theory may be more applicable. The experimental results for oxygen and nitrogen seem to differ even more widely from the corresponding results obtained by theory.

SUMMARY AND CONCLUSION.

A quartz tube of the transparent variety was found to be pervious to hydrogen at temperatures ranging from 330° C. to 710° C., and at pressures varying from 20 cm. mercury below to 20 cm. mercury above atmospheric pressure. In the case of oxygen and nitrogen, no leakage could be detected for pressures less than one atmosphere. Nitrogen did not seem to escape until a temperature of about 430° C. was obtained. At constant temperature, the results for all gases showed considerable increase in leakage with increasing pressures. Under approximately like conditions, hydrogen leaks most rapidly and nitrogen least rapidly.

The results may possibly be explained by assuming that if the quartz contains occluded gas, that this gas is in equilibrium and forms a constituent part of the quartz itself. It is reasonable to believe that this occluded gas does not effect the process of diffusion in any other manner.

The writer wishes to acknowledge his indebtedness to Prof. J. S. Shearer, at whose suggestion the work was begun, and particularly to Prof. E. G. Merritt, who has aided and encouraged the work in every possible way. Valuable assistance has been received from Dr. R. P. Anderson, Dr. E. H. Kennard and Prof. F. R. Sharpe.

THE GEIGER APPARATUS FOR THE PHOTOGRAPHIC REGISTRATION OF ALPHA AND BETA PARTICLES.

BY J. E. SHRADER.

IN 1908 Rutherford and Geiger[1] devised an electrical method for counting alpha particles given off by radio-active substances. The method may be briefly described as follows: Through a small opening the alpha particles were allowed to enter a strong electric field between a wire and a coaxial cylinder. The entrance of each single particle produced ions, which in this strong electric field gave rise to a current by collision between the wire and the cylinder. The current, thus produced, was detected by a quadrant electrometer, which was connected to the wire while the cylinder was connected to the negative terminal of a high potential battery. The gas in the cylinder was carbon dioxide at reduced pressure.

The difficulty encountered in this experiment was that ionization or "natural disturbance," as the authors call it, arose from the walls of the cylinder, as they explain, and gave a deflection of the electrometer when no alpha particles entered the vessel. By using a cylinder of small diameter and carbon dioxide at reduced pressure they found that these disturbances could be made very small.

In order that a greater number of particles might be counted Rutherford and Geiger[2] devised another form of apparatus to be used with a string electrometer and a photographic registration apparatus.

The ionization chamber was a cylinder 3 cm. long with a hemispherical end. The rays entered the chamber through a small mica window in the hemispherical end. A rod with a spherical knob was insulated in the axis of the cylinder. This rod was connected to a string electrometer which was connected to earth through a high resistance of alcohol and xylol. A high negative potential was applied to the cylinder. The gas used in the chamber was helium at about 20 cm. pressure. This gas greatly decreased the deflections due to "natural disturbances."

Later, Geiger[3] used a fine point inside a cylindrical ionization chamber and was able to get a discharge at atmospheric pressure with a positive

[1] Proc. Roy. Soc. (A), 81, 141, 1908.

[2] Phil. Mag. (6), 24, 618, 1912.

[3] Deut. Phys. Gesell., 15, 534, 1913.

potential of about 1,200 volts applied to the cylinder. The cylinder was 2 cm. in diameter and 4 cm. long. The end of the cylinder was closed except for an opening 1.4 mm. in diameter through which the rays entered. The point which was a fine steel needle extended to within .8 cm. of the end. The difficulty encountered was to find a point which was free from "natural disturbances." The choice of a point was made on examination with a microscope. The point was cleaned with alcohol and if it gave "natural disturbances" it was heated in the Bunsen flame. This treatment often removed the difficulty.

An investigation along the line of Rutherford and Geiger's original experiment was made by Myssowsky and Nesturch.[1] They were not able to secure conditions which were free from "natural disturbances." Their conclusions were that the method was untrustworthy and could not be used to determine with accuracy the number of particles given off by radioactive substances.

At the suggestion of Professor Pegram, of Columbia University, the author began an investigation of the conditions for readily getting the result observed by Geiger.

In these experiments an ionization chamber of the same type as that used by Geiger was employed. Not having a string electrometer at

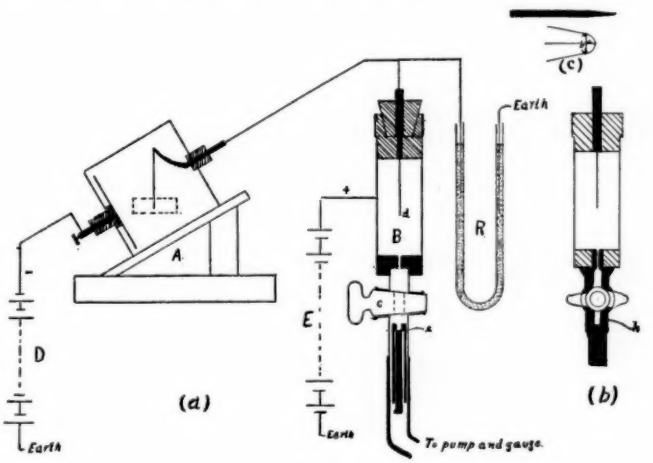

Fig. 1.

my command, I made a Wilson tilted electrometer, which, with a liquid resistance of alcohol and xylol served the purpose very well. With this arrangement thirty or forty deflections per minute could easily be counted. A diagram of the arrangement is shown in Fig. 1, *a*.

[1] Ann. der Phys., 43, 461, 1914.

Much difficulty was experienced in finding points which were free from "natural disturbances." These "natural disturbances" are shown by a deflection of the gold-leaf of the electrometer when no rays enter. These disturbances may be intermittent or continuous. When they are intermittent they give rise to a sudden deflection of the gold-leaf. This is due to its gaining a charge which immediately leaks off through the high resistance bringing the leaf again to its zero position. If the disturbances are continuous the leaf remains deflected as long as the discharge continues. When a deflection is due to an entering particle the fact can be determined by removing or screening the source of radiation, upon which the leaf will remain quiet. To distinguish the discharge caused by an entering ray from the discharge caused by a "natural disturbance," I will refer to it as the *ray discharge*.

In the preliminary work, long slender needles were used as points. The smoothest and finest points were selected on examination with a microscope. These were carefully cleaned with alcohol and if they gave "natural disturbances" they were carefully heated in the Bunsen flame. Occasionally this would remedy the difficulty. It was observed that best results were obtained by heating just enough to blue the point. If the point was heated until it turned gray it could not be made to give the ray discharge. Careful heating in the oxidizing flame of the blast lamp always gave best results.

Among a number of samples of needles two types of points were found. One type has a rounding slope toward the end, which, as seen through the microscope, seems very blunt. The other kind, called "sharps" by the manufacturer, has two slopes toward the point, the last slope bringing the needle to an extremely sharp point. Most slender needles have the first kind of point. Even among the kind called "sharps," the finer needles do not have better points than those which are coarser.

A dozen of number 9 "sharps" were selected to be tested. Of these, five gave the ray discharge while the others gave only "natural disturbances." The other remaining seven were heated by the blast lamp and five of these were made good, so that only two out of the twelve failed altogether. However none were entirely free from disturbances and these became very numerous if the voltage applied to the cylinder was increased over a short range. These twelve needles were now laid carefully away exposed to the air of the room. Four days later they were again tested under the same conditions. Only one, and this one had been previously heated by the blast lamp, would now give the ray discharge. Four days later they were tested again. The one previously in good order now failed, but two of the others gave the ray discharge. Upon reheating all except one were made good.

From the foregoing results it would appear that the points were continuously undergoing a change in their surfaces. This might be due to collection of dust particles, condensation of water vapor, or the formation of a gas layer. To investigate this point further it seemed desirable to use a platinum point so that it could be heated without destroying the shape of the point or roughening the surface. Accordingly a platinum wire (no. 20, B. & S.) was pointed by grinding on an emery stone and turned true in a jeweler's lathe with a smooth fine-grained stone until it was fairly sharp. The finishing was done by hand on the stone. In this way a good point, even sharper than could be found on the best needles, was made.

All points at first gave natural disturbances. These were now heated in the blast lamp at intervals and tested. The disturbances became less frequent as the heating proceeded and after prolonged heating ceased altogether. Dipping the point while red-hot into nitric acid seemed to help remove the disturbances. Many times when the point gave a good deflection for the ray discharge I have watched for ten minutes at a time without detecting a single disturbance. Occasionally, on standing for some time with no potential applied some disturbances would appear. These disturbances may be due to occlusion of gases, condensation of moisture on the point or the collection of dust particles. The two latter causes seem the more obvious since the introduction of fresh air into the apparatus often results in disturbances. These disturbances, however, disappear upon reheating and dipping into nitric acid.

If a fresh point is heated just enough for the ray discharge to appear it is more sensitive than if heated thoroughly. This sensitiveness gradually decreases until in the course of a half of an hour it is about one sixth of its former value. The change in the sensitiveness now changes more slowly and finally reaches a very small value. When heated thoroughly the sensitiveness of the point remains constant, no change being observed during the greater part of a day.

If, in any circumstance, a spark discharge passed inside the cylinder, the point always developed disturbances. This is undoubtedly due to the deposit of a foreign substance on the point during discharge, though in such a small quantity that it could not be detected by the microscope. Reheating and dipping into nitric acid usually removed the difficulty, though sometimes a repolishing of the point was necessary.

USE OF DIFFERENT MATERIALS AS POINTS.

Various metals were used as points. Among these were, copper, brass, aluminum, silver, iron and 20 k. gold. All of these oxidize on

heating. Though many attempts were made to get the ray discharge from these, none were successful except in the case of silver. A successful silver point was always more difficult to obtain than a platinum point so that no further trials were made with the other metals. Undoubtedly the reason a steel needle gave the ray discharge and the pointed iron wire would not was owing to the polish of the surfaces. This would result in unequal oxidation upon heating, in the case of the hand-pointed points.

THE EFFECT OF THE SHARPNESS OF THE POINT.

A sharpening of the point always results in a lowering of the potential to produce a constant deflection for the ray discharge. A well-seasoned platinum point was used. It was first made quite blunt. It was then heated and tested, noting the potential necessary to give a deflection of ten scale divisions in the telescope. The point was gradually made sharper, then it was heated again and tested under the same conditions. The diameter, $b = 2a$, as shown in Fig. 1, c, was measured with a microscope. The results of the variation of the diameter of the point with the potential of the point necessary to give a constant deflection of ten scale divisions are shown in Fig. 2. It can be seen that as the point becomes very sharp the potential drops very rapidly.

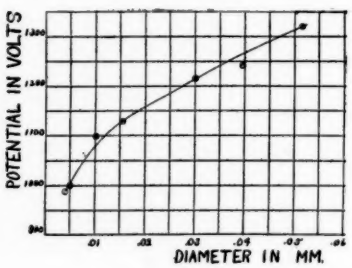

Fig. 2.

THE EFFECT OF THE SIZE OF THE CYLINDER.

In Rutherford and Geiger's original experiment they gave as the cause of the "natural disturbances" the radiations from the sides of the chamber. They found that if a smaller cylinder was used these disturbances became smaller and less frequent. In their second experiment they used helium in the chamber and largely got rid of these disturbances. To test the effect of the size of the cylinder, several cylinders varying in diameter from .9 cm. to 5.3 cm. were made. The result showed that with a thoroughly seasoned point one size of cylinder was as free from disturbances as another. The only difference observed was in the potential to give a ray discharge. In the case of the largest cylinder, the point had to be brought quite close to the end to produce the ray discharge with the potential, 1,400 volts, which I had at my command.

A piece of apparatus similar to that used by Rutherford and Geiger in their original experiment was now made. A small platinum wire was stretched centrally in the axis of the cylinder and fastened to larger brass wires passing through ebonite ends in the cylinder.

While the air was being pumped out an electric current was sent through the wire until it became incandescent. The large brass wires did not extend through to the inside of the chamber so that the extreme ends of the platinum wire which would not be heated to incandescence were not exposed inside the cylinder. When tested before heating the wire, with the chamber partially exhausted, both with the cylinder at positive and then at negative potential, numerous natural disturbances were given off. As the heating proceeded these disturbances became less numerous but could not be removed entirely even though heated for many minutes. The heating was stopped as the wax which secured the ends began to melt. Finally the wire was removed from the cylinder, cleaned and thoroughly heated in the blast lamp and replaced. Now the highest potential (+ 1,400 volts) which I had available failed to show any disturbance or to give the ray discharge though the pressure was reduced to a few millimeters of mercury. This part of the work was not pursued further.

These observations show that the seat of the disturbances is in the point or central wire and that they are not due to the chamber or the contained gas provided it is dry and dust free. This latter will be more fully shown in later observations.

THE EFFECT OF VARIOUS GASES IN THE CYLINDER.

To study the effect of various gases in the chamber the apparatus shown in Fig. 1, *a*, was used. The platinum point was firmly held in the ebonite end by a brass rod. A one-hole rubber stopper slipped over this rod and fitted into the ebonite end. By this arrangement the point was held firmly in place, the chamber was made air-tight and the point could easily be removed. The other end of the cylinder was closed with a metallic end into which was sealed a glass stopcock in which was placed the active rod, *e*. The rays could be cut off by closing the cock. Through the cock the chamber could be exhausted and various gases introduced. Connected to the stopcock was also the pump and pressure gauge.

The gases used were air, oxygen, hydrogen and carbon dioxide. Observations were taken by varying the pressure and observing the potential necessary to give a constant deflection of ten divisions with Alpha rays. The observations were taken with the point at negative and then at positive potential.

Air from the room was at first used. Then the effect of drying it by bubbling it through sulphuric acid and passing it through a calcium chloride tube was observed. Drying the air makes an increase in potential necessary to secure a constant deflection. This is more noticeable at the higher pressures. The introduction of moist air always produced natural disturbances. All other gases were carefully dried. The same point was used throughout these observations. Before a new gas was introduced the point was removed and reheated.

When the pressures were low a small change in potential often resulted in a spark discharge. In this case the point had to be removed and reheated. Many times at the lower pressures disturbances entered which were entirely absent at higher pressures. The number of these were small in comparison to the number of ray discharges so that no attempt was made to get rid of them.

No noticeable difference in the operation of the apparatus was apparent whether the point was negative or positive except that in all cases the potential had to be raised when the point was positive. From the appearance of the curves in Fig. 3 this difference tends to disappear at

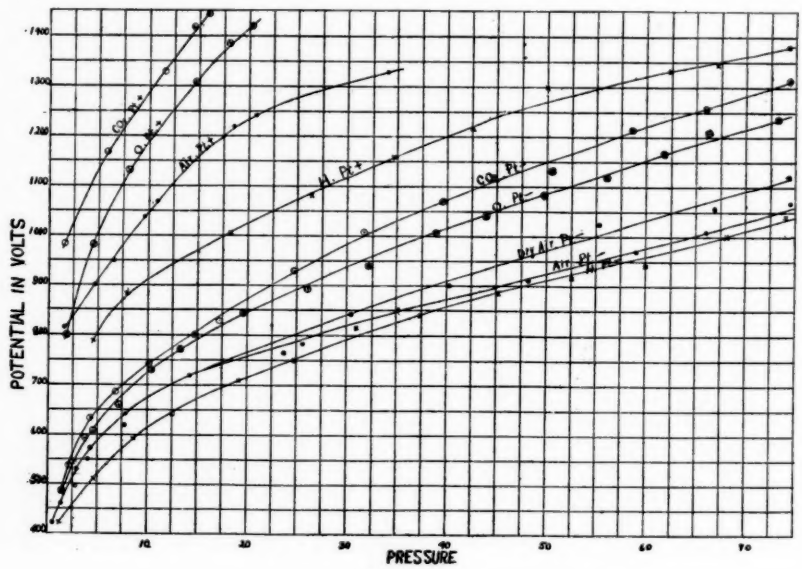

Fig. 3.

very low pressures as the curves tend to converge toward the origin. The difference in the negative and the positive potential for the ray discharge for hydrogen is much less marked than for any other gas. The potentials for constant deflection at a certain pressure increases with the density of the gas.

THE EFFECT OF GEOMETRICAL DISTRIBUTION OF THE PARTICLES INSIDE THE IONIZATION CHAMBER.

In order that Geiger's method may be effective in determining the number of alpha or beta particles given off by radioactive substances it is quite necessary to know that every particle that enters the chamber is registered. Scattering always occurs when alpha or beta rays penetrate matter or pass through a gas. By scattering in the gas in the cylinder the rays may be distributed at varying distances from the point. The question to be settled is how close to the point must the particle come in order to be registered. To determine this the cylinder previously used, was fitted with a brass end containing several openings 1.4 mm. in diameter distributed at varying distances from the center. These openings were made of larger diameter on the outside so that a brass tube containing a rod made active by exposure at negative potential to thorium emanation could be inserted, the source of radiation being three centimeters from the opening. The distance of the point from the opening was varied from 10 to 25 mm. The active rod was inserted in the openings at various distances from the center of the end. Under these conditions the number of deflections for five minutes was noted. An ebonite end similar to the brass end was now used and similar observations taken. The voltage used throughout was 1,200 volts. The magnitude of deflection did not differ much in the two arrangements when the point was farther from the end. When the point was near the end the deflections were larger with the brass end than with the ebonite end. The results are shown in the following table:

The active rod did not have the same strength in the two cases, so that no comparison can be made as to the number of alpha particles registered by the two arrangements.

These results show that the field from the end of the cylinder to the point is stronger near the center, so that when rays enter to one side, only those that are projected in a favorable direction are registered. If all rays enter this strongest part of the field they are very apt to be registered. If the point should happen to be displaced even a little from the center the rays could not enter the strongest part of the field and would not all be registered. This was tested, and it was found that a small displacement of the point from the center resulted in a diminution of the number of deflections, especially when the point was near the end. With an ebonite end the rays enter a fairly uniform field so that all are more apt to be registered. It is seen that as the point is removed farther from the brass end the number registered is practically the same whether the rays enter at the center or to one side. This last statement is practically

L = length of cylinder; d = distance of the point from the end; r = the distance from the center of the opening through the particles entered; N = number of deflections in five minutes.

Brass End.				Ebonite End.			
L	d	r	N	L	d	r	N
4 cm.	25 mm.	0 mm.	37	4 cm.	25 mm.	0 mm.	46
"	"	2	34	"	"	2	44
"	"	3	34	"	"	3	41
"	"	4	25	"	"	4	43
"	"	5	29	"	"	5	45
3.5	20	0	37	3.5	20	0	43
"	"	2	36	"	"	2	43
"	"	5	35	"	"	3	41
				"	"	4	46
3	15	0	37	"	"	5	40
"	"	2	30				
"	"	3	18	3	15	0	45
"	"	4	14	"	"	2	40
"	"	5	3	"	"	3	39
				"	"	5	42
3	10	0	37				
"	"	2	6	3	10	0	46
				"	"	3	40
				"	"	5	38

true in the case of an ebonite end for all distances of the point from the end.

In Geiger's experiment he placed the point 8 mm. from the end. Since this produces a less uniform field, one would not expect to get the same size deflections for all particles, hence at the smaller potentials the smaller discharge would not be shown. That this is true is shown in his table of results; for the potential has to be increased considerably above the voltage at which the ray discharge begins to take place before the number registered becomes constant. With an ebonite end the number registered at all potentials is constant. To show the truth of the last statement some further observations were made, using the chamber shown in Fig. 1, *b*. The point was kept at negative potentials varying from 1,020 to 1,400 volts. Unfortunately I did not have a stronger source of radiation so that a greater number of deflections could be made per minute. In each case the number of deflections in five minutes was observed. The deflection of the electrometer was not quite proportional to the potential to which the leaf was raised, hence the scale of the telescope was calibrated in terms of volts applied to the leaf and the deflections recorded were made numerically equal to the

voltage of the leaf. The instrument was made less sensitive for the higher potentials applied to the point. The results are shown in the following table:

Alpha Rays.

Volts.	Deflection.	Number Observed in Five Minutes.
1,020	13	87
1,070	24	85
1,120	35	82
1,180	58	88
1,230	64	87
1,300	86	87
1,340	100	85
1,400	140	81

Beta Rays.

Volts.	Deflection.	Number Observed in Five Minutes.
1,220	18	32
1,280	26	31
1,340	43	31
1,400	62	30

This table shows that the number of particles registered remains practically constant for all voltages. In Fig. 4, curve 1 shows the vari-

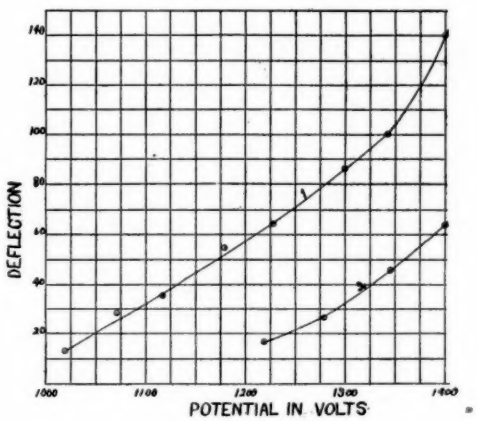

Fig. 4.

ation of deflection with voltage. Curve 2 shows this variation for beta rays.

The highest potential to which the leaf of the electrometer was raised was 140 volts. This was obtained with 1,400 volts applied to the point. This was the highest potential I had available. Judging from the form of the curve, which is an ionization by collision curve, the application

of higher potential might raise the leaf to a potential nearly twice as high before a spark discharge would occur.

USE OF THE APPARATUS FOR DECAY MEASUREMENTS.

The apparatus was now used to determine the rate of decay of thorium B by counting the number of alpha particles given off by its products. The value found was 10.4 hours. Considering the fact that the source of radiation was weak, this is in very good agreement with the accepted value, 10.6 hours.

CONCLUSIONS.

By the use of a platinum point which can be thoroughly heated without destroying the point the "natural disturbances" can all be driven out so that without any difficulty a point can be secured which will give only the ray discharge.

If protected from moist fresh air "natural disturbances" show no tendency to develop during the use of the apparatus.

The seat of the "natural disturbances" is in the point and their number is not affected by the size of the ionization chamber.

At constant pressure, the potential at which the ray discharge occurs is a function of the sharpness of the point.

With a thoroughly heated platinum point, air, hydrogen, oxygen and carbon dioxide, if thoroughly dried, can be used equally well in the cylinder. The only difference is one of the value of the potential necessary to produce the ray discharge.

No more difficulty is experienced in using the point at positive potential than at negative potential. When the point is positive the higher potential is necessary.

With an ebonite end in the ionization chamber quite uniform deflections are obtained for entering rays. As a result, the number of particles registered is constant for all potentials of the point for discharges which give measureable deflections.

The apparatus has been used to determine the rate of decay of thorium B and the agreement with the accepted value is good.

The simplicity of the apparatus and the certainty of the results when a thoroughly heated platinum point is used makes it adaptable to many kinds of radioactive measurements.

WILLIAMS COLLEGE,
May 4, 1915.

A MECHANICAL PROCESS FOR CONSTRUCTING HARMONIC ANALYSIS SCHEDULES FOR WAVES HAVING EVEN AND ODD HARMONICS.

BY HAWLEY O. TAYLOR.

PERIODIC curves or waves may be analyzed into their constituent harmonics by a variety of methods, some of which make use of mechanical analyzers,[1] some use integration and others measured ordinates.[2] The labor involved when a mechanical analyzer is not at hand is considerable if a complete analysis is desired. Simple methods have been devised for waves which have odd harmonics only, so that schedules for computing the amplitude and phase of the harmonics may be constructed by a mechanical process. The object of this paper is to outline a simple, mechanical process for constructing complete schedules for the analysis of waves containing even, as well as odd, harmonics, and also to point out how a single schedule may be made for any particular harmonic.

Following is the table of symbols which will be used:

$A_1 \cdots A_{n-1}$ = amplitudes of the sine components of first, $\cdots$ $(n-1)$st harmonics.....cm.
A_k = amplitude of the sine component of the kth harmonic.....cm.
α = time phase.....radians.
B_0 = distance through which axis of periodic curve is removed from coördinate axis....cm.
$B_1, \cdots B_n$ = amplitudes of the cosine components of first, $\cdots$ nth harmonics.....cm.
B_k = amplitude of the cosine component of the kth harmonic.....cm.
C = amplitude of resultant wave.....cm.
k = order of a particular harmonic in a wave.....numeric.
λ = serial number of a particular ordinate or division point on the abscissa of the wave.....numeric.
n = number of parts in a half wave-length.....numeric.
$\omega = (2\pi/\tau)$ = angular velocity.....radians/second.
t = time elapsed from a given epoch.....seconds.
t_λ = time elapsed between y_0 and y_λ.....seconds.
τ = periodic time.....seconds.
$u_1, u_2, \cdots$ = sums of pairs of ordinates.....cm.
$v_1, v_2, \cdots$ = differences of pairs of ordinates.....cm.
y = ordinate of periodic wave, or displacement in a wave motion.....cm.
$y_1, y_2, \cdots y_n$ = lengths of the first, second and nth ordinates of periodic curve.....cm.
y_λ = length of a particular ordinate, λ.....cm.

The equation of a periodic curve representing a wave containing harmonics may be written:

[1] Bibliography 1–9.
[2] Bibliography 10–27.

$$y = \sum_{k=1}^{n-1} A_k \sin k\omega t + B_0 + \sum_{k=1}^{n} B_k \cos k\omega t \tag{1}$$

$$= A_1 \sin \omega t + A_2 \sin 2\omega t + \cdots + B_0 + B_1 \cos \omega t + B_2 \cos 2\omega t + \cdots \tag{2}$$

$$= C_1 \sin (\omega t + \alpha_1) + C_2 \sin (\omega t + \alpha_2) + \cdots + B_0 \tag{3}$$

The analysis consists in finding the coefficients, A and B, there being n of each, making $2n$ unknown quantities to be computed. The A's and B's are then used to find the C's and α's of the final equation, which give the amplitude and phase of each harmonic of the curve.

To compute these coefficients, divide the wave-length, 2π, into $2n$ equal parts from the origin of coördinates on the abscissa, and measure the lengths, $y_1, y_2, \cdots y_{2n}$, of the ordinates erected at the division points.

Take, as an example, the case of $n = 12$. The interval between ordinates is $2\pi/2n = 360°/24 = 15°$.

The values of the 24 ordinates (each with its proper algebraic sign) are arranged in the following scheme and the sums and differences taken:

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}
	y_{24}	y_{23}	y_{22}	y_{21}	y_{20}	y_{19}	y_{18}	y_{17}	y_{16}	y_{15}	y_{14}	y_{13}
Sums,	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}
Differences,		v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	

where

$$u_0 = y_{24}, \quad u_{12} = y_{12}, \quad u_{11} = y_{11} + y_{13}, \quad v_1 = y_1 - y_{23}.$$

A schedule of the v's is made for computing the values of the A-coefficients by placing, in a column at the left, the sines of the angular distances of the ordinates from the origin and putting in the v's as shown below:

In the same way, for computing the B-coefficients, the u's are placed according to the following schedule:

To compute the A's and B's, the numbers represented by the u's and v's are multiplied by the sine of the angle at the left, and the sum of each group taken. This sum is equal to the quantity ($12A_1$ etc.) below each group. From the equations thus established, the coefficients are found. The calculation is shown by the following example:

$$12A_3 = (v_4 + v_8) \sin 0° + (v_1 + v_3 - v_5 - v_7 + v_9 + v_{11}) \sin 45° + (v_2 - v_6 + v_{10}) \sin 90°.$$

B_0 is found from the relation:

$$24B_0 = u_0 + u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7 + u_8 + u_9 + u_{10} + u_{11} + u_{12}.$$

Schedules for A-coefficients.

sin	$12A_1$	$12A_2$	$12A_3$
0		$+v_6$	$+v_4 \quad +v_8$
15	v_1+v_{11}		
30	v_2+v_{10}	$v_1+v_5-v_7-v_{11}$	
45	v_3+v_9		$v_1+v_3-v_5-v_7+v_9+v_{11}$
60	v_4+v_8	$v_2+v_4-v_8-v_{10}$	
75	v_5+v_7		
90	v_6	$v_3 \quad -v_9$	$v_2 \quad -v_6 \quad +v_{10}$

sin	$12A_4$	$12A_5$	$12A_6$
0	$v_3 \quad +v_6 \quad +v_9$		$v_2+v_4+v_6+v_8+v_{10}$
15		v_5+v_7	
30		v_2+v_{10}	
45		$-v_3-v_9$	
60	$v_1+v_2-v_4-v_5+v_7+v_8-v_{10}-v_{11}$	$-v_4-v_8$	
75		v_1+v_{11}	
90		v_6	$v_1-v_3+v_5-v_7+v_9-v_{11}$

sin	$12A_7$	$12A_8$	$12A_9$
0		$v_3 \quad +v_6 \quad +v_9$	$v_4 \quad +v_8$
15	v_5+v_7		
30	$-v_2-v_{10}$		
45	$-v_3-v_9$		$v_1+v_3-v_5-v_7+v_9+v_{11}$
60	v_4+v_8	$v_1-v_2+v_4-v_5+v_7-v_8+v_{10}-v_{11}$	
75	v_1+v_{11}		
90	$-v_6$		$-v_2 \quad -v_6 \quad -v_{10}$

sin	$12A_{10}$	$12A_{11}$
0	v_6	
15		v_1+v_{11}
30	$v_1+v_5-v_7-v_{11}$	$-v_2-v_{10}$
45		v_3+v_9
60	$-v_2-v_4+v_8+v_{10}$	$-v_4-v_8$
75		v_5+v_7
90	$v_3 \quad -v_9$	$-v_6$

For checks, we have but to remember that, for $t = 0$, $y = y_0$ or y_{2n}, for $t = \tau/4$, $y = y_{n/2}$, for $t = \tau/2$, $y = y_n$ and for $t = 3\tau/4$, $y = y_{3n/2}$.

Putting these values of t into equation (2), we have:

$$y_{2n} = B_0 + B_1 + B_2 + \cdots + B_n,$$
$$y_n = B_0 - B_1 + B_2 - \cdots + B_n,$$
$$y_{n/2} = A_1 - A_3 + A_5 - \cdots - A_{n-1} + B_0 - B_2 + B_4 - \cdots + B_n,$$
$$y_{3n/2} = -A_1 + A_3 - A_5 + \cdots + A_{n-1} + B_0 - B_2 + B_4 - \cdots - B_n.$$

For the case of $n = 12$, the checks are, therefore:

$$y_{24} = u_0 = B_0 + B_1 + B_2 + B_3 + B_4 + B_5 + B_6 + B_7 + B_8 + B_9 + B_{10} + B_{11} + B_{12},$$
$$y_{12} = u_{12} = B_0 - B_1 + B_2 - B_3 + B_4 - B_5 + B_6 - B_7 + B_8 - B_9 + B_{10} - B_{11} + B_{12},$$

Schedules for B-coefficients.

sin 0	u_6	$u_3 + u_9$	$u_2 + u_6 + u_{10}$
15	$u_5 - u_7$		
30	$u_4 - u_8$	$u_2 - u_4 - u_8 + u_{10}$	
45	$u_3 - u_9$		$u_1 - u_3 - u_5 + u_7 + u_9 - u_{11}$
60	$u_2 - u_{10}$	$u_1 - u_5 - u_7 + u_{11}$	
75	$u_1 - u_{11}$		
90	$u_0 - u_{12}$	$u_0 - u_6 + u_{12}$	$u_0 - u_4 + u_8 - u_{12}$
	$12B_1$	$12B_2$	$12B_3$

sin 0		u_6	$u_1+u_3+u_5+u_7+u_9+u_{11}$
15		$u_1 - u_{11}$	
30	$u_1-u_2-u_4+u_5+u_7-u_8-u_{10}+u_{11}$	$u_4 - u_8$	
45		$-u_3 + u_9$	
60		$-u_2 + u_{10}$	
75		$u_5 - u_7$	
90	$u_0 - u_3 + u_6 - u_9 + u_{12}$	$u_0 - u_{12}$	$u_0-u_2+u_4-u_6+u_8-u_{10}+u_{12}$
	$12B_4$	$12B_5$	$12B_6$

sin 0	u_6		$u_2 + u_6 + u_{10}$
15	$-u_1 + u_{11}$		
30	$u_4 - u_8$	$-u_1-u_2-u_4-u_5-u_7-u_8-u_{10}-u_{11}$	
45	$u_3 - u_9$		$-u_1+u_3+u_5-u_7-u_9+u_{11}$
60	$-u_2+u_{10}$		
75	$-u_5 + u_7$		
90	$u_0 - u_{12}$	$u_0 + u_3+u_8 + u_9 + u_{12}$	$u_0 - u_4+u_8-u_{12}$
	$12B_7$	$12B_8$	$12B_9$

sin 0	$+ u_3 + u_9$	u_6	
15		$- u_5 + u_7$	
30	$u_2 - u_4 - u_8 + u_{10}$	$u_4 - u_8$	
45		$- u_3 + u_9$	
60	$- u_1 + u_5 + u_7 - u_{11}$	$u_2 - u_{10}$	
75		$- u_1 + u_{11}$	
90	$u_0 - u_6 + u_{12}$	$u_0 - u_{12}$	
	$12B_{10}$	$12B_{11}$	

$$y_6 = A_1 - A_3 + A_5 - A_7 + A_9 - A_{11} + B_0 - B_2 + B_4 - B_6 + B_8 - B_{10} + B_{12},$$

$$y_{18} = -A_1 + A_3 - A_5 + A_7 - A_9 + A_{11} + B_0 - B_2 + B_4 - B_6 + B_8 - B_{10} + B_{12}.$$

A_1 and B_1 are the amplitudes of the sine and cosine components of the fundamental or harmonic of the first order. The amplitude and phase of this first harmonic are found by combining these two component waves into one wave, given by

$$y = C_1 \sin(\omega t + \alpha_1),$$[1]

[1] Bibliography 29.

having an amplitude

$$C_1 = \sqrt{A_1^2 + B_1^2},$$

and a phase displacement

$$\alpha_1 = \tan^{-1}(B_1/A_1).$$

The amplitude and phase of all other harmonics are found in the same way, but, in plotting the curves, the phase displacement, α, must be measured to the scale of the wave-length of the particular harmonic; *i. e.*, in the case of the third harmonic, say; the phase displacement is represented by a distance one third as long as that which gives it for the first harmonic, etc. In general, for the kth harmonic,

$$y = C_k \sin(k\omega t + \alpha_k),$$

where

$$C_k = \sqrt{A_k^2 + B_k^2},$$

and

$$\alpha_k = \tan^{-1}(B_k/A_k),$$

and for plotting

$$\text{length } \alpha_k = (1/k)\tan^{-1}(B_k/A_k),$$

the scale of the fundamental being used for all harmonics.

C may be taken as always positive and α given the sign of B. Then $+\alpha$ is measured in the negative direction from the point taken as the origin of coördinates for all of the curves, and $-\alpha$ is measured in the positive direction. The value of α found is always 90° or less, but α is less than 90° only when A is positive, and is greater than 90° when A is negative, or

$$\text{for } +A, \quad \alpha < 90°$$

and

$$\text{for } -A, \quad \alpha > 90° \ (= 180° - \alpha).$$

In dividing the abscissa into $2n$ parts for analysis, the origin of coördinates may, of course, be taken at any point along the wave. C_k and the relative phase displacement of the different harmonics are unaffected by the position chosen, but the numerical value of α_k depends upon the location of this point.

A glance at the schedules for A and B reveals a simple, mechanical process by which they may be constructed.

MECHANICAL PROCESS FOR CONSTRUCTION OF SCHEDULES.

The smaller the number of parts into which the wave-length is divided, the simpler will be the work, but, for accuracy, n should be chosen larger than the highest significant harmonic present. If the presence of the 11th harmonic is suspected, then let $n = 12$ (or any larger number which will divide evenly into 360). For the angular interval between

ordinates, divide the wave-length, 360°, by $2n$ (in this case, 24); thus, $360°/24 = 15°$. The sines of these angular intervals are marked in consecutive order in a column at the left, beginning with sin 0° and ending with sin 90°.

The values of the u's and v's are found from the sums and differences of the measured ordinates (y_1, y_2, etc.) according to the scheme already given. (In this scheme, the ordinates for the first half wave-length are placed from left to right on the upper line and for the second half, from right to left on the lower line, the lower line being shifted one place to the left, as shown.)

For A_k, the v's are placed in consecutive order, *down and up*, etc., in each kth space beginning with the kth space below sin 0° and changing sign each time of passing through sin 0°. The subscripts of the v's are from 1 to 11, [$11 = (n - 1)$], inclusive.

For B_k, the u's are placed in consecutive order, *up and down*, etc., in each kth space beginning with sin 90°, where u_0 is placed, and changing sign each time of passing through sin 0°. The subscripts of the u's are from 0 to 12, ($12 = n$), inclusive. The sign of both u and v is positive at the beginning.

This process may be illustrated for the ninth harmonic, say, by letting $k = 9$. Then, for the A-schedule, count 9 *down and up*, beginning with *one* at sin 15°; *two* will be at sin 30°, *six* at sin 90°, *seven* at sin 75° and *nine* at sin 45° where $+ v_1$ is placed; then, for the second 9, we have *one* at sin 30°, *three* at sin 0° where the sign changes from + to −, and *nine* at sin 90° where $- v_2$ is placed; etc. The count is continued until $+v_{11}$ is written down.

For the B-schedule, begin at sin 90°, where $+ u_0$ is placed; then count *one* at sin 75°, *six* at sin 0° where the sign changes from + to −, and *nine* at sin 45° where $- u_1$ is placed; the count is then continued and the u's placed in the same manner as described for the v's until $- u_{12}$ is written down.

Below the group for the A-coefficient is written $12A_9$ and for the B-coefficient, $12B_9$. This completes the schedules. For the general case, nA_k and nB_k are written below each group.

The illustration given also explains how a single schedule is made for any particular harmonic. The harmonic for which a schedule is made may be of much lower order than the highest significant harmonic present in the wave but n should be chosen larger than this highest harmonic, as already pointed out.

MATHEMATICAL DISCUSSION.

The opening paragraphs of this discussion are a translation of the comprehensive treatment given in Orlich's book.[1]

"In general, a periodic curve is represented only by a Fourier's series of an infinitely large number of terms, but in most practical cases the terms of higher order are so small that they may be neglected. We may assume that the curve may be represented with sufficient accuracy by a series of $2n$ coefficients, and may let

$$y = \sum_{k=1}^{n-1} A_k \sin k\omega t + B_0 + \sum_{k=1}^{n} B_k \cos k\omega t. \tag{1}$$

"In order to find the coefficients, A and B, we divide the period, τ, into $2n$ parts from the origin of coördinates on the abscissa, and measure the lengths, $y_1, y_2, \cdots, y_{2n}$, of the ordinates erected at the division points. Moreover, the ordinate, y_λ, belongs to the abscissa, t_λ $(= \lambda\tau/2n)$. These corresponding values, y_λ and t_λ, must satisfy equation (1), *i. e.*,

$$y_\lambda = \sum_{k=1}^{n-1} A_k \sin k\omega t_\lambda + B_0 + \sum_{k=1}^{n} B_k \cos k\omega t_\lambda$$

for $\lambda = 1, 2, \cdots 2n$.

"There are $2n$ linear equations from which the $2n$ unknowns, $A_1, \cdots A_{n-1}, B_0, B_1, \cdots B_n$, may be calculated. The calculation gives the result:

$$nA_k = \sum_{\lambda=1}^{2n} y_\lambda \sin k\omega t_\lambda \tag{4}$$

$$nB_k = \sum_{\lambda=1}^{2n} y_\lambda \cos k\omega t_\lambda \tag{5}$$

$$\Big\} \text{ for } k = 1, 2, \cdots (n-1).$$

For $k = 0$ and $k = n$, the two sums will equal, respectively, $2nB_0$ and $2nB_n$. There is also the relation:

$$k\omega t_\lambda = (2k\pi/\tau)(\lambda\tau/2n) = k\lambda\pi/n. \tag{6}$$

"The correctness of the formulæ, (4) and (5), is established as soon as we allow the number, $2n$, of division points on the abscissa to become infinitely large.[2] Thereby, the infinite summations go over into the integral formulæ:

$$A_k = (2/\tau)\int_0^\tau y \sin k\omega t\,dt$$

$$B_k = (2/\tau)\int_0^\tau y \cos k\omega t\,dt$$

$$\Big\} \text{ for } k = 1, 2, 3, \cdots."$$

These formulæ are the well-known expressions for the coefficients in the Fourier's series.

In order that convenient schedules may be made for use in the analysis of waves which include both odd and even harmonics, a simplification may be affected by combining terms which are removed an equal distance on each side of the middle ordinate $(\lambda = n)$. The expressions for two such ordinates, except for the sign of the sine-term, are equal, as seen from the following:

$$\begin{aligned} \sin k\omega t_{2n-\lambda} &= \sin [\{k(2n - \lambda)\pi\}/n] = \sin [2k\pi - k\pi\lambda/n] \\ &= \sin 2k\pi \cos (k\lambda\pi/n) - \cos 2k\pi \sin (k\lambda\pi/n) \\ &= -\sin (k\lambda\pi/n) \\ &= -\sin k\omega t_\lambda, \end{aligned}$$

[1] Bibliography 9.

[2] Bibliography 28.

$$\begin{aligned}\cos k\omega t_{2n-\lambda} &= \cos [\{k(2n-\lambda)\pi\}/n] = \cos [2k\pi - k\lambda\pi/n] \\ &= \cos 2k\pi \cos (k\lambda\pi/n) + \sin 2k\pi \sin (k\lambda\pi/n) \\ &= \cos (k\lambda\pi/n) \\ &= \cos k\omega t_\lambda.\end{aligned}$$

Combining these results with equations (4) and (5), we have:

$$nA_k = \sum_{\lambda=1}^{2n} y_\lambda \sin k\omega t_\lambda = \sum_{\lambda=1}^{n-1} (y_\lambda - y_{2n-\lambda}) \sin (k\lambda\pi/n), \tag{7}$$

$$nB_k = \sum_{\lambda=1}^{2n} y_\lambda \cos k\omega t_\lambda = \sum_{\lambda=1}^{n-1} (y_\lambda + y_{2n-\lambda}) \cos (k\lambda\pi/n) + y_{2n} + y_n \cos k\pi. \tag{8}$$

The last two terms in (8) correspond to $\lambda = n$ and $\lambda = 0$ or $2n$, which are not included elsewhere in the summations. They are found by putting $\lambda = n$ and $\lambda = 2n$ in (4) and (5).

The construction of schedules from formulæ (7) and (8) follows:

For the case of $n = 12$, the interval between the ordinates, $2\pi/2n$, is 15°. The values of the 24 ordinates are written in two lines as in the scheme given above and the sums, u, and the differences, v, taken as called for in (7) and (8).

Substituting the u's and v's in equations (7) and (8):

$$\begin{aligned}nA_k &= v_1 \sin 15k + v_2 \sin 30k + v_3 \sin 45k + v_4 \sin 60k + v_5 \sin 75k \\ &\quad + v_6 \sin 90k + v_7 \sin 105k + v_8 \sin 120k + v_9 \sin 135k \\ &\quad + v_{10} \sin 150k + v_{11} \sin 165k. \\ nB_k &= u_0 + u_1 \cos 15k + u_2 \cos 30k + u_3 \cos 45k + u_4 \cos 60k \\ &\quad + u_5 \cos 75k + u_6 \cos 90k + u_7 \cos 105k + u_8 \cos 120k \\ &\quad + u_9 \cos 135k + u_{10} \cos 150k + u_{11} \cos 165k + u_{12} \cos 180k.\end{aligned}$$

By transforming all the angle functions into sines of angles lying between 0° and 90°, and arranging in a column according to angle, the schedules already given are obtained. Each line is multiplied by the sine of the angle set down in the left-hand column. The summations of the groups containing v are, in each case, equal to nA_k, k being the order of the respective harmonic, and of those containing u, nB_k.

$B_0 = (1/2n)\sum_{\lambda=1}^{2n} y_\lambda$, from which the value of B_0 already given is obtained. If the y's are not measured from the axis of the curve, but from a line parallel to it and a distance B_0 from it, the schedules for A and B may still be applied as they stand, for, in that case, each y is changed by an amount equal to B_0, and the way in which the + and − signs are distributed among the y's for each coefficient, causes the B_0 to cancel out in the summations.

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THE CHANGE IN THE ELASTICITY OF ALUMINUM WIRE WITH CURRENT AND EXTERNAL HEATING.

BY H. L. DODGE.

THIS paper is the third of a series upon the effect of temperature on the elasticity of wires and deals with aluminum. The apparatus is the same as that used in work with copper and mild steel wires already described in the PHYSICAL REVIEW.[1] Upon these metals very exhaustive tests were made, the necessity of this being evident from the great variation in the results that had been reported. The treatment of the specimens of aluminum and the methods employed to secure a wide variation in the conditions have been the same as are fully described in former papers. In general the results have been the same. Consequently this paper is restricted to a brief discussion of the effects peculiar to aluminum and to a discussion of certain general conclusions regarding the effect of temperature on the elasticity of metals.

The tests have been made upon two samples of aluminum wire known commercially as "soft" and "hard." The former is annealed wire and bends easily, the latter is unannealed and is stronger and stiffer. The wires and their chemical analyses were secured through the kindness of Mr. H. M. Hall, superintendent of the United States Aluminum Co., Massena, N. Y. Below are tabulated the analyses, certain physical constants and data regarding the tests.

Chemical Analyses and Constants.

	Soft Aluminum.	Hard Aluminum.
Iron	.44 per cent.	.32 per cent.
Silicon	.24 " "	.22 " "
Copper	.03 " "	.07 " "
Aluminum	99.29 " "	99.39 " "
Diameter	1.24 mm.	1.44 mm.
Breaking load	22. kg.	38. kg.
Coefficient of expansion	.000025	.000025
Length	57.7 cm.	57.5 cm.
Permanent load	1,114. g.	1,114. g.
Added load	1,048. g.	1,048. g.
Total load per sq. mm.	1.8 kg.	1.3 kg.
Maximum current	23. amp.	
Maximum current per sq. mm.	19.1 amp.	

[1] PHYS. REV., N. S., Vol. 2, 431, 1913; Vol. 5, 373, 1915.

Aluminum will not stand very heavy loads. The percentage accuracy of the determinations of Young's modulus is consequently not as high as has been the case when the loading and resulting total stretch were considerably greater. The curves may however be regarded as giving the change in the modulus to an accuracy of one per cent. The error in the measurement of the temperature is of course greatest at high temperatures but probably does not exceed ten or at the most fifteen degrees.

TESTS UPON SOFT ALUMINUM WIRE.

The specimen of soft aluminum wire was first subjected to loads of two and four kilograms and a temperature of about 250° C. This treatment straightened out the kinks and lengthened the wire several millimeters. The weights were then made of appropriate sizes and two series of readings were taken with external heating. Series 2, Fig. 1,

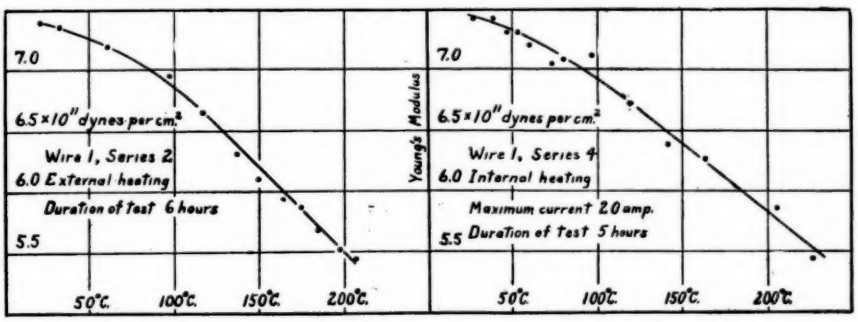

Fig. 1.

Effect of current and external heating upon the Young's modulus of a soft aluminum wire.

shows how the rate of decrease of the modulus increased rapidly with increase of temperature until it became uniform at about 100° C. The third series was with decreasing temperature and was similar in character. Series 4, Fig. 1, differs only in the manner of heating, an electric current in the wire itself being used.[1]

A comparison of these results with those shown in series 5, Fig. 2, shows that a permanent change in the properties of the specimen resulted from the treatment in test 4. In this test the temperature reached was higher than before, resulting in some stretching of the wire. The annealing was consequently more thorough. Series 5 was with external heating, increasing and decreasing temperature, and may be regarded as giving the change of Young's modulus with temperature for a thoroughly annealed aluminum wire.

[1] In all figures dots represent observations with increasing temperature, crosses those with decreasing temperature.

Series 6 was taken in the same manner except that the heating was by a current in the wire itself. The curves in the two drawings are practically the same. They show that a cyclic state had been reached in which Young's modulus becomes a function of temperature and is

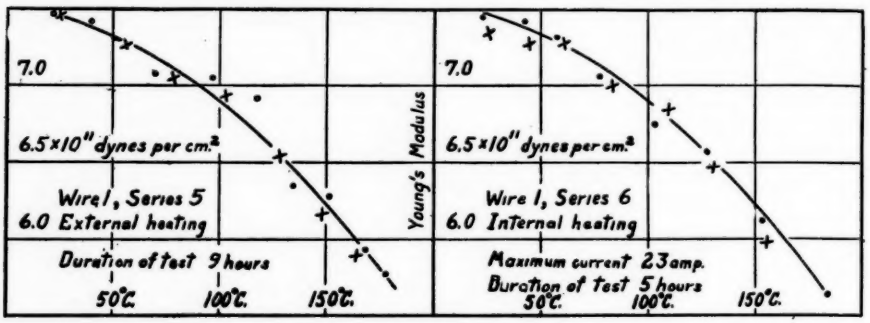

Fig. 2.

Effect of current and external heating upon the Young's modulus of a soft aluminum wire after thorough annealing.

independent of the manner of heating and the thermal route by which any temperature is reached.

Table I. gives the complete data from which series 6 is plotted. This is characteristic of all the data, except that with external heating a longer time was necessary for temperature equilibrium to be reached.

Tests upon Hard Aluminum Wire.

The sample of hard aluminum wire was carefully straightened and then tested without any preliminary annealing. Series 1, Fig. 3, shows the almost linear relation between the modulus and temperature. This

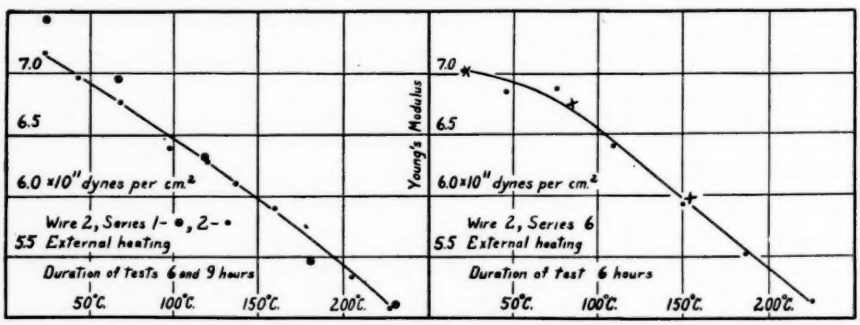

Fig. 3.

Effect of temperature upon the Young's modulus of a hard aluminum wire.

changes slightly with the second heating as shown by the curve of series 2. The next few series of observations were taken under a variety

of conditions, some with heavier weights and at higher temperatures. Just before series 6 the wire was heated to about 400° C. This treatment resulted in a partial annealing and produced the change in the properties of the wire shown by Fig. 3. The similarity of these results to those secured with the soft aluminum specimen lead me to believe that further testing with the consequent thorough annealing would have developed the same final conditions as before.

TABLE I.

Data for Soft Aluminum Wire, Series 6, Internal Heating.

Obs. No.	Time.	Temp.	Stretch.	Current.	No. of Obs.	Young's Modulus $\times 10^{-11}$, Dynes per cm.^{2}
1	9:00 A.M.	22° C.	.0658 mm.	0 amp.	9	74.5
2	10:00	47	.0660	8	10	74.2
3	10:20	57	.0670	11	9	73.1
4	10:35	78	.0694	14	8	70.6
5	10:50	102	.0726	17	10	67.5
6	11:00	128	.0745	19	11	65.8
7	11:15	154	.0800	21	6	61.2
8	11:25	183	.0870	23	23	56.4
9	11:30	156	.0820	21	7	59.8
10	11:40	131	.0758	19	13	64.6
11	11:50	110	.0717	17	10	68.4
12	12:05 P.M.	83	.0700	14	9	70.0
13	1:00	61	.0673	11	9	72.9
14	1:20	44	.0674	8	11	72.8
15	1:45	25	.0668	0	10	73.4

The results of the tests upon samples of soft and hard aluminum wire may be summarized as follows:

1. Aluminum wire can by thorough annealing be brought to a cyclic condition or steady elastic state in which Young's modulus becomes a function of temperature.[1]

2. Within given temperature limits the modulus becomes practically independent of history, the thermal route by which any temperature is reached having no apparent effect on the value of the modulus.

3. Heating by an electric current has no effect other than that caused by the accompanying temperature.

4. The Young's modulus of annealed wire decreases with increase of

[1] Guthe and Sieg, PHYS. REV., Vol. 30, 610, 1910, and Sieg, PHYS. REV., Vol. 31, 421, 1910, in studying the elastic constants of platinum-iridium wires by means of torsional vibrations found that annealing would bring the wires to such a state that results were easily reproducable.

temperature at an increasing rate. The following table is compiled from the results for thoroughly annealed soft aluminum wire.[1]

TABLE II.

Change of Young's Modulus of a Thoroughly Annealed Aluminum Wire with Increase of Temperature.

Temperature.	Young's Modulus $\times 10^{-11}$, Dynes per cm.^{2}	$\frac{dE \times 10^{-11}}{dT}$	Total Decrease, Per Cent. of E at 20° C.
20° C.	7.50	.0046	0.0
40	7.40	.0057	1.3
60	7.27	.0072	3.1
80	7.10	.0090	5.3
100	6.90	.0112	8.0
120	6.66	.0135	11.2
140	6.38	.0155	14.9
160	6.06	.0165	19.2
180	5.72	.0170	23.8

5. Hard aluminum wire exhibits an almost constant temperature coefficient of elasticity. Annealing does not materially change the value of Young's modulus but it does however affect the rate of change, increasing the temperature coefficient of elasticity for low temperatures and decreasing it for higher values, the dividing temperature being about 100° C.

GENERAL CONCLUSIONS REGARDING THE EFFECT OF TEMPERATURE UPON THE YOUNG'S MODULUS OF METALS.

When the present series of investigations on the effect of current and external heating upon the Young's modulus of metals was undertaken the literature of the subject revealed great inconsistency and lack of uniformity in the results that had been secured. Although most observers had reported a decrease of elasticity with increase of temperature many did not find this to be the case. So great were the variations in the results that no conclusion could be drawn regarding the true nature of the effect of temperature on Young's modulus.

Two explanations were possible. Either every specimen possessed its own peculiar elastic properties or else the apparatus and methods employed were not sufficiently refined for the very delicate measurements required.

[1] Slotte, Acta. Soc. Scien. Fennicæ, Vol. 26, 1899, for a temperature range of 6° C. to 70° C. reports an almost uniform decrease in the modulus of .16 per cent. per degree. Katzenelsohn, Diss. Berlin, 1887, reports a decrease of .19 per cent. per degree between 0° C. and 100° C.

It was my belief that samples of wire ought by the proper heat treatment to be brought to a condition in which the modulus would be a function of temperature. With this as a criterion the apparatus described in previous papers was developed. The results secured with the cruder forms at once convinced me that a great part at least of the inconsistencies in former work must have been due to poor temperature control and the intrusion of certain factors affecting the accuracy of the determination of the modulus. The greatest difficulties were met when the heating was by an electric current in the wire itself as might have been anticipated from the fact that no satisfactory investigations had ever been carried out with this method of heating.

Copper was chosen for the first tests because of its high electrical conductivity and with the idea that it ought to respond readily to moderate heat treatment. My first paper[1] describes how the copper wire finally reached a cyclic state in which the Young's modulus became a function of temperature, the modulus decreasing at an increasing rate. The experience gained in the study of this metal lead me to believe that there was no reason why other metals should not behave in a similar manner. If any metal were to put this idea to test it seemed as if it would be one possessing such variable and peculiar properties as iron or steel. The second paper[1] describes the results with a mild steel wire. A cyclic state was secured for a temperature range of 20° C. to 475° C. and the general nature of the change of the modulus with temperature proved to be the same as in the case of copper.

Since these experiments were performed almost identical results have been reported by Lea and Crowther[2] with rods of mild steel, nicro-copper, and high-tension brass and by Harrison[3] with nickel wire, although in the latter case anomalous effects occur near the critical temperature. Similar results are also to be found in the work of some of the earlier investigators.

In the present paper aluminum has been shown to yield exactly the same type of results as were secured with copper and mild steel. A considerable amount of data is now available and for the metals that have been mentioned the following generalizations appear to be justified:

1. By thorough annealing the metal can be brought to a cyclic state in which Young's modulus becomes a function of the temperature.

2. The Young's modulus of the metal decreases with increase of temperature, the rate of the change increasing with the temperature.

[1] *Loc. cit.*

[2] Engineering, Vol. 98, 487, 1914.

[3] Phys. Soc., Proc., Vol. 27, 8, 1914.

3. When the metal is heated by the passage of an electric current the change in the modulus is a pure temperature effect.

In my own mind it seems so reasonable to expect that other metals will behave in the same manner that I am led to predict that these generalizations will be found to be the laws governing the effect of temperature upon the elasticity of metals.[1] Work is being continued in this laboratory with other metals and it is to be hoped that other investigators will continue with different types of apparatus and under different conditions.

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May, 1915.

[1] Dodge, PHYS. REV., N. S., Vol. 5, 76, 1915.